(1.1) Number, operation, and quantitative reasoning. The student uses whole numbers to describe and compare quantities.	1.1A: The student is expected to compare whole numbers to 99 (less than, greater than, or equal to) using sets of concrete objects and pictorial models.				
Materials: Base-ten blocks (9 tens rods and 9 ones	units)				
Procedure: Student will use base ten blocks to deten Numbers should vary in complexity, in order to disco value (e.g., 45 and 54; 75 and 85; 40 and 50, etc.). Al	over more about the student's understanding of place				
Arrange two groups of base ten blocks to represent two numbers. The tens rods should be placed next to each other standing vertically and the ones units should be arranged to the right of the tens rods. Vary the arrangements so the smaller number is not always on the left or right.					
Point to the set of blocks that shows the greater (bigger) number. Or: Point to the set of blocks that shows the smaller number. Or:					
Tell me which number is larger.					
If the numbers represented are equal, the student should respond that they are the same. After identifying which number is greater or lesser, record correct and incorrect responses below.					
Check Student's Response:	Check Student's Strategies:				
 1. Numbers & □ Correct □ Incorrect 	 Counted by tens and ones Counted only by ones Did not count but only pointed to one set Lined up the blocks to compare them Other: 				
 2. Numbers & □ Correct □ Incorrect 	 Counted by tens and ones Counted only by ones Did not count but only pointed to one set Lined up the blocks to compare them Other: 				
Notes:					

1.1A: The student is expected to compare whole numbers to 99 (less than, greater than, or equal to)	Possible interpretations, issues to follow up on, and implications for teaching
using sets of concrete objects and pictorial models.	

This task requires students to use their understanding of place value to compare the value of numbers. Pay attention to the student's errors, if any, and plan further questions and teaching strategies to address these errors.

- The student who **counts by tens and ones** demonstrates an understanding of place value and how it can be used to identify values represented by the sets of blocks. A teaching strategy might include working with larger numbers, perhaps introducing the hundreds cube as an additional challenge.
- A student who **counts all of the individual units in the tens and ones piles** may not yet have a clear understanding of the value of the tens place. A teaching strategy might include working only with ones units and adding a certain number of unit cubes to the ones pile, then asking the student what should be done if there are more than ten in the pile. Also, provide practice in counting by tens.
- A student who is **determining values based on a visual assessment of the block sets** may be counting them in her head or may be making a decision based on what looks like more or less. In this case, it is important to ask the student how she knows which one is bigger. This will provide more information about her understanding of place values.
- Lining up the blocks in order to compare value is a good strategy but may only provide the student with a visual understanding of place value. It is important to support this strategy by connecting the physical materials with their numerical equivalents. A teaching strategy might include providing the student with paper to write the numbers under each block set in order to help associate the blocks with the numerals. After viewing the numbers, the student may be able to compare them without lining up the blocks.

(1.1) Number, operation, and quantitative reasoning. The student uses whole numbers to describe and compare quantities.1.1B: The student is expected to create sets of and ones using concrete objects to describe, compare, and order whole numbers	tens			
Materials: Base ten blocks (tens, ones), two mats (different colors)				
Procedure: Student will use base ten blocks to represent one number on each mat. Use numbers that vary in complexity (e.g., 45 and 54; 75 and 85; 40 and 50, etc.)	t			
Please use these blocks to show me [number] on this mat. Now please use the blocks to show me [number] on this mat.				
When student completes both number sets: Can you tell me how these two block sets are alike or different?				
Circle Student's Response:				
1. Numbers &				
Response: Correct Incorrect Counted: by tens and ones by ones on	ly			
Confused tens and ones places Other:				
2. Numbers &				
Response: Correct Incorrect Counted: by tens and ones by ones on	ly			
Confused tens and ones places Other:				
3. Numbers &				
Response: Correct Incorrect Counted: by tens and ones by ones on	ly			
Confused tens and ones places Other:				
4. Numbers &				
Response: Correct Incorrect Counted: by tens and ones by ones on	ly			
Confused tens and ones places Other:				
Repeat this task with other numbers as needed.				
Notes:				

1.1B: The student is expected to create sets of tens	Possible interpretations, issues for follow up,
and ones using concrete objects to describe,	and implications for instruction
compare, and order whole numbers.	

This task requires students to use their understanding of place value to both create and compare block sets according to their values. Pay attention to the student's errors, if any, and plan further questions and teaching strategies to address these errors.

- The student who **counts by tens and ones** demonstrates an understanding of place value and how it can be used to identify values represented by the sets of blocks. A teaching strategy would include working with larger numbers, perhaps introducing the hundreds cube as an additional challenge.
- A student who **counts all of the individual units in the tens and ones piles** may not yet have a clear understanding of the value of the tens place. A teaching strategy would include working only with ones units and adding a certain number of unit cubes to the ones pile, then asking the student what should be done if there are more than ten in the pile. Also, provide practice in counting by tens.
- If the student **has difficulty aligning the blocks with the numeral they represent**, a teaching strategy would include simplifying the numbers presented. Working with smaller numbers or those with zero in the ones place may help him or her understand which blocks represent which place value (e.g., the number 50 is represented with 5 tens rods and no ones units).

(1.1) Number, operation, and quantitative reasoning. The student uses whole numbers to describe and compare quantities.	1.1C: The student is expected to identify individual coins by name and value and describe relationships among them.			
Materials: Four coins, or pictures of coins, heads up (quarter, dime, nickel, penny)				
Procedure: Show student four coins.				
Can you tell me the names of each of these coins? What is the value of this coin? (point to each one in Which coin has the greatest value? Which coin has the least value? Which coins have a value more than the nickel? Which coins have a value less than the dime? How many of this coin (point to the nickel) make up How many of this coin (point to the penny) make up	p the value of this coin? (point to the dime)			

A			
Check Student's Response:			
1. Identify quarter by name	6. Identify which coin is worth the least		
\Box Correct	□ Correct		
	□ Other		
	7. Identify which coins are worth more than the		
2. Identify dime by name	nickel		
□ Incorrect	□ Other		
3. Identify nickel by name	8. Identify which coins are worth less than the dime		
□ Correct	□ Correct		
□ Incorrect	□ Other		
4. Identify penny by name	9. Identify nickels in a dime		
□ Correct	□ Correct		
	□ Other		
5. Identify which coin is worth the most	10. Identify pennies in a dime		
□ Correct	□ Correct		
□ Other	□ Other		
Notes:			

	1 2	Possible interpretations, issues to follow up on, and implications for teaching
among them.	-	

This task will provide you with information regarding your students' understanding of the names and values of individual coins, as well as the relationships among them. This knowledge will develop as students have more exposure to coins and using money. A student who is aware of the names of the coins but not their values may need more exposure.

- Often, as students are learning the values, **they confuse the dime and the nickel**, associating the greater size of the nickel with a greater value. (Other currencies do not do this.) A teaching strategy might be to have students work together in pairs with a coin game that reinforces the values of each coin (e.g., choosing a dime, nickel, or penny from a bag of coins and moving a marker the corresponding number of spaces).
- Students who have learned the names and values and understand the relationships among the coins may be ready to begin combining coins to make different values. A teaching strategy might involve providing opportunities for students to "make purchases" using combinations of coins (e.g., setting up a "store" where students may purchase stickers or crayons for prices that require different combinations of coins). (Remember, coin combinations are a Grade 2 expectation so such activities wouldn't be appropriate for all students).

(1.1) Number, operation, and quantitative reasoning. The student uses whole numbers to describe and compare quantities.1.1D: The student is expected to read and write numbers to 99 to describe sets of concrete objects.				
 Materials: Base 10 blocks (or whatever your class uses to represent units), including groups of 10 units (e.g., single straws might represent units, bundles of 10 straws, groups of 10 units) Pencil and paper 				
 Procedure: I'm going to show you some blocks (or whatever you're using) and ask you what number the blocks represent. Then I want you to write that number on this paper. Let's try one together. Show student 7 unit blocks. For example, if I ask you what number these blocks show, you would say '7' and write the number '7' on this paper. Now you try. 				
Place within reach of student the blocks that represent the numbers: 23, 51, 67, 80, 99 (one number at a time). For example, for 23, the child sees two tens sticks and three units.				
Place groups of 10 units and single units in front of student from right to left. What number do these blocks show? (<i>Wait for response.</i>) Write that number on this paper.				
 Check Student's Verbal Response: Correct: Student correctly reads the numbers represented by each block construction (e.g., says 23 for 2 ten rods and 3 one rods) Incorrect: Student incorrectly reads the numbers (e.g., "two, three,") Student responds with a number other than what is represented:				
 Check Student's Strategies: Counted each block as one unit (e.g., student counted each ten block as one unit and each unit block as one unit). Counted units one-by-one (e.g., rather than counting 10 blocks by 10, student counts '1,2,3,10') Counted blocks by groups (e.g., student counts 10 blocks by tens and single units one-by-one: '10, 20, one, two three') Miscounts (e.g., 23= '10, 20, 21, 22, 23,24') Other: 				

1.1D: The student is expected to read and write numbers to 99 to describe sets of concrete objects.

Possible interpretations, issues to follow up on, and implications for teaching

- If the student **counted each block as one unit**, does the student understand that the number 10 can be represented with a numeral in the tens place? A teaching strategy might include training in base ten and in place value. A possible activity could involve trading or exchanging ten single units for a representation of a group of ten. Another activity is relating the groups of ten to numerals in the tens place.
- If the student **counted the blocks correctly, but rather than counting by 10s, she or he counted the ten units represented by the 10s blocks one-by-one,** "1, 2, 3...10," does the student understand that it is possible to count representations of 10 by 10s? A goal might be to teach students to count by 10s and/or provide students with practice counting by 10s.
- If the student **counted blocks by groups** (e.g., student counts 10 blocks by tens and single units one-by-one: "10, 20, one, two, three") she is probably well on her way to understanding place value. A teaching strategy might involve extending this knowledge to work with written numerals. Ask the student to write a numeral such as 306, and a) ask the student to write the numeral that is one less than 306; b) ask the student to write the numeral that is 10 less than then 306; c) 10 more than 306; and d) 100 less than 306.
- If the student **miscounted**, why do you think that was? Is she just sloppy or not paying attention, or not skilled at counting? If you believe sloppiness might be responsible, a goal might be to teach students ways to check their work. An activity that could be useful might involve pairing students and asking them to check one another's work (how diligent they become when the work being checked is someone else's!).
- If the student **wrote the number incorrectly**, why do you think that was? For example, if the student wrote "203" for "23" did the student make the mistake of writing "203" because that is what the number sounds like phonetically in English? What about if the student wrote "32" for "23"? Was it because the student didn't understand that the number of 10 blocks should have been written in the 10s place and the ones in the 1s place? Or did the student frequently make unintended reversals? If you believe the student is not well practiced in writing symbolic representations of numbers, a teaching goal might be to provide the student with such practice. This might involve working with number lines (perhaps the student would benefit from having one at her disposal to consult as needed), and identifying and practicing writing conventionally written numbers found in diverse environmental print (on cereal boxes, measuring tools, calendars, etc.)

(1.3) **Number, operation, and quantitative reasoning.** The student recognizes and solves problems in addition and subtraction situations. **1.3B:** The student is expected to use concrete and pictorial models to apply basic addition and subtraction facts (up to 9+9=18 and 18-9=9).

Materials: Counters (counting bears, or chips, or similar materials), Base ten blocks, paper and markers

Procedure: Students will answer basic <u>addition</u> problems up to 9+9=18 using their model of choice. Place the materials in front of the student and tell him or her that any material can be used to solve the problem. Vary the order of numbers presented for addition. Sometimes the larger should be first and sometimes the smaller. Optional approaches include questions using doubles (e.g., 5+5), patterns (e.g., 4+3, 4+4, 4+5), and plus one (5+1, 6+1), etc.

It is not necessary to present a large number of problems at once, but by presenting them by increasing levels of difficulty, you will be able to document student progress.

What is [number] plus [number]?

If the student does not use a concrete or pictorial model to solve the problem, ask the student to show how to solve the problem with one of the concrete models provided or by drawing a picture.

(Prompt if no model used) **Please show me how you solved the problem with one of these** [point to manipulatives] **or by drawing a picture.**

Cir	cle Student's Response:	Circle Stude	ent's Model:
1.	Numbers +	Concrete:	fingers counters Base-10 blocks
	Response: Correct Incorrect	On Paper:	drawings tallies numerals
	Addition Strategy: count all count on	Other:	
	Other:		nodel until prompted
2.	Numbers +	Concrete:	fingers counters Base-10 blocks
	Response: Correct Incorrect	On Paper:	drawings tallies numerals
	Addition Strategy: count all count on	Other:	
	Other:		nodel until prompted
3.	Numbers +	Concrete:	fingers counters Base-10 blocks
	Response: Correct Incorrect	On Paper:	drawings tallies numerals
	Addition Strategy: count all count on	Other:	
	Other:		nodel until prompted
Rep	beat this task with other numbers as needed.		

Notes:

1.3B: The student is expected to use concrete and pictorial models to apply basic addition and subtraction facts (up to 9+9=18 and 18-9=9).

Possible interpretations, issues for follow up, and implications for instruction.

This task requires students to solve basic addition problems using their model of choice. Take note of the student's choice of model(s) and addition strategies. Pay attention to the student's errors, if any, and plan further questions and teaching strategies to address these errors.

The student used fingers or counters (concrete models):

- If the student **counted all**, a teaching strategy might include more practice with simple addition problems using concrete models to become more fluent with computations and to learn to count on with numbers other than one. If the response was incorrect, follow up with additional problems to determine where the student is struggling. Have the child show you how he or she solved the problem with counters and ask him or her to point while counting.
- If the student **counted on**, try some more challenging addition problems with concrete models and also encourage him or her to try using pictorial models or symbols to represent his or her thinking. You may also want to begin encouraging him or her to use the Base ten blocks to represent addition problems with larger numbers.

The student used **Base ten blocks** (concrete model):

- If the student used both rods and units, but used them all as individual **counters**, see section above (for **fingers** or **counters**). This student does not yet have an understanding of how Base ten blocks represent the base ten system and may not even understand the base ten system.
- If the student used only the unit blocks, find out if he or she understands that the blocks have different values by asking why there are different sizes and shapes. If the student doesn't know why there are different sizes/shapes, see section above (for **fingers** or **counters**). However, if the student is able to communicate that the different blocks represent different values, the student may have some emerging understanding of the base ten system as represented by the Base ten blocks. Try asking the student to represent some larger numbers using the Base ten blocks. A teaching strategy might then include instruction on representing larger numbers using the Base ten blocks or presenting more challenging addition problems and asking the student to solve them with the Base ten blocks.
- If the student used both rods and units appropriately, making exchanges between units and rods, this student demonstrates an understanding of the base ten system. A teaching strategy might include having the student solve problems with larger numbers with Base ten blocks, or problems using symbols to scaffold the development of an algorithm.

The student used **drawings** or **tallies** (pictorial models):

• Although these models appear to be abstract, the student's understanding of addition may still be limited. See section above (for **fingers** or **counters**) for general teaching strategies based on student's addition strategy. However, utilize pictorial models instead of concrete models, where appropriate, and encourage movement towards symbolic models.

The student used **numerals (symbolic model):**

- If the student solved the problem correctly, have him or her explain how he or she solved the problem. Explore the child's mental strategies that accompany the symbolic model. Is the student visualizing concrete materials? Is he or she then counting all or counting on? Have him or her demonstrate his or her strategy using concrete or pictorial models. Is he or she using memorized or derived facts? If he or she is visualizing concrete materials, a teaching strategy might include more practice with similar addition problems to encourage greater fluency and the development of more efficient mental strategies.
- If the student solved the problem incorrectly, have him or her explain how he or she solved the problem. Explore the child's mental strategies. If the student realizes his or her mistake while explaining the solution, see the section for correct solutions above. If the student doesn't realize his or her mistake, try to identify where the student's strategies begin to fail. Have the student use concrete or pictorial models to check his or her work.
- If the student **did not use concrete or pictorial models** initially, how did he or she figure out the solution? Did he or she count silently? Did he or she visualize concrete materials? Ask him or her how he or she solved the problem mentally. See section above (for **numerals**) for general teaching strategies.

(1.4) **Patterns, relationships, and algebraic thinking.** The student uses repeating patterns and additive patterns to make predictions.

1.4: The student is expected to identify, describe, and extend concrete and pictorial patterns in order to make predictions and solve problems.

Materials:

- About 20 blocks or unifix cubes
- 6 cards, each cut to about the same size as the cubes being used.

Procedure: Watch me very closely. I'm going to make a special kind of pattern.

Within the student's view, construct an additive pattern like this, leaving 'blank' spaces (cards) as shown in the picture below:				
A B C D E Place three cards before the first column and 3 car picture above. They are labeled in the picture for s should go in the empty spaces marked by cards				
Procedure:		Check Studer	nt's R	Response:
Point to the space being held by card D.				
How many blocks should go here?		8 (Correct)		Other:
Show me how many go here.		Places 8 blocks (Correct)		Another number of blocks:
Why?		Reasonable response		No response, or unreasonable
Point to the space being held by card C.				
How many blocks should go here?		3 (Correct)		Other:
Show me how many go here.		Places 3 blocks (Correct)		Another number of blocks:
Why?		Reasonable response		No response, or unreasonable
Point to the space being held by card F				
How many blocks should go here?		10 (Correct)		Other:
Show me how many go here.		Places 10 blocks (Correct)		Another number of blocks:
Why?		Reasonable response		No response, or unreasonable
Point to the space being held by card A				
How many blocks should go here?		1 (Correct)		Other:
Show me how many go here.		Places 1 blocks (Correct)		Another number of blocks:
Why?		Reasonable response		No response, or unreasonable

Check Student's Strategies:

How many blocks should go here?

- □ The student generally counted each column individually ("This is the third column"), but did not necessarily count the number of blocks within each column.
- \Box The student generally counted aloud each column and each block within each column.

Show me how many blocks go here.

- \Box The student generally counted out a group of blocks, then stacked them in the empty space.
- □ The student generally stacked blocks one-by-one into the empty space until it "looked right."

Why?

- \Box The student generally provided a reasonable explanation for why a certain number of blocks are needed to continue the pattern.
- \Box The student generally did not provide a reasonable explanation.

Notes:

1.4: The student is expected to identify, describe, and extend concrete and pictorial patterns in order to make predictions and solve problems.

Possible interpretations, issues to follow up on, and implications for teaching

How many blocks should go here?

- If the student simply **counted each column individually** ("This is the seventh column"), but did not necessarily count the number of blocks within each column, why might that be? Was the child unaware of the fact that the number of blocks in each column was part of the linear growing/shrinking pattern? A teaching goal might be to challenge the student to build and describe "staircase" patterns.
- If the student **counted aloud each column and each block within each column and correctly predicted how many blocks should go in the empty space**, the student might be ready to explore even more complex patterns. Introduce the student to patterns that grow by twos or threes and/or try hiding elements in different places of pattern and ask the student to predict what is hidden.

Show me how many blocks go here.

- If the student was **able to place the correct number of blocks to continue the pattern**, do you think he or she understood the rule of the pattern? If so, perhaps the student is ready to work with even more complex growing patterns. A teaching strategy could involve challenging the student to build his or her own pattern with his or her own rules. Activities might include instructing students to build patterns different from the ones you showed them.
- If the student did **correctly continue the pattern**, why might that be? Was she simply 'eyeballing' the pattern, placing one block on top of another in the fifth column until the column looked to be a bit higher than the preceding column? If the student did not seem to grasp the rule of this pattern, perhaps she would be better served exploring simpler patterns. Construct a simple AB pattern with something such as blue and red counting bears. Repeat the AB core at least 4 times then cover one bear in the pattern with a paper cup and ask the student, "What goes here?" Make the task more difficult by covering more than one bear or by making a more complex pattern, such as ABB.

Why?

- If the student **provided a reasonable explanation** for why a certain number of blocks are needed to continue the pattern, a teaching strategy might involve challenging the student to analyze and communicate rules of even more complex patterns. Challenge students to build their own complex patterns, hide elements, and ask peers to predict the identities of hidden elements.
- If the student **did not explain herself or himself**, a teaching strategy could include challenging the student to describe his or her thought process to you in various contexts. Continue to ask this student questions such as "How did you know?" so that you can better understand his or her thinking and he or she can practice communicating his or her ideas.

(1.5) Patterns, relationships, and algebraic thinking. The student recognizes patterns in numbers and operations.	1.5A: The student is expected to use patterns to skip count by twos, fives, and tens.			
Materials: About 20 blocks or unifix cubes				
Procedure:				
Within the student's view, construct an additive pattern for	r each of the following questions:			
Let's try counting in different ways.				
Can you count by twos for me, like this: 2, 4, 6? Now you try.				
Can you count by fives for me, like this: 5, 10, 15? Now you try.				
Don't use blocks for this one:				
Can you count by tens for me, like this: 10, 20? Now yo	ou try.			
Record the highest number student counts to.				
Check Student's Response: By twos: Highest correct number				
□ Has difficulty with changes in tens place (e.g., 22, 24, 26, 28?)				
\Box Skips numbers in the pattern (e.g., 12, 14, 18)				
□ Not able to continue pattern				
□ Other:				
By fives: Highest correct number				
\Box Has difficulty with changes in tens place (e.g., 20, 25, ?,?)				
\Box Skips numbers in the pattern (e.g., 45, 55, 60)				
□ Not able to continue pattern				
□ Other:				
By tens: Highest correct number				
□ Has difficulty with changes in tens place (e.g., 10, 20 ? ?)				
\Box Skips numbers in the pattern (e.g., 10, 20, 40)	□ Skips numbers in the pattern (e.g., 10, 20, 40)			
□ Not able to continue pattern				
□ Other:				
Notes:				

1.5A: The student is expected to use patterns to	Possible interpretations, issues to follow up on,
skip count by twos, fives, and tens.	and implications for teaching

This task requires students to use their understanding of number patterns to skip count by tens, fives, and twos. This is challenging for students who have not yet mastered the patterns. A teaching strategy would involve using a highlighted 100s chart for the different patterns to help the student learn the pattern and gain confidence.

Students who have **mastered these patterns** may enjoy the challenge of saying every other number in the pattern (e.g., playing a game with a friend where they alternate saying numbers: child one says "5," child two says "10," child one says "15," etc.)

(1.5) Patterns, relationships, and algebraic thinking. The student recognizes patterns in numbers and operations.		1.5B: The student is expected to find patterns in numbers, including odd and even.	
Materials: 100s charts with different patterns of numbers highlighted (odds, evens, fives, tens)			
Procedure:			
Present different hundreds charts to students and ask them to identify the patterns represented by the highlighted numbers			
Can you describe the pattern created by the numbers in red?			
Check Student's Response:	Check	x Student's Strategies:	
Odds		Counted highlighted numbers out loud	
□ Correct		Pointed to highlighted numbers	
□ Other:		Counted numbers between highlighted numbers	
		Identifies visual pattern (e.g., "they make lines up and down")	
		Other:	
Evens		Counted highlighted numbers out loud	
□ Correct		Pointed to highlighted numbers	
□ Other:		Counted numbers between highlighted numbers	
		Identifies visual pattern	
		Other:	
Fives		Counted highlighted numbers out loud	
□ Correct		Pointed to highlighted numbers	
□ Other:		Counted numbers between highlighted numbers	
		Identifies visual pattern	
		Other:	
Tens		Counted highlighted numbers out loud	
□ Correct		Pointed to highlighted numbers	
□ Other:		Counted numbers between highlighted numbers	
		Identifies visual pattern	
		Other:	

1.5B: The student is expected to find patterns in numbers, including odd and even.

Possible interpretations, issues to follow up on, and implications for teaching

This activity assesses students understanding of number patterns. While many students may initially only recognize the visual patterns made when highlighting numbers in the 100s chart, this may eventually lead to an understanding of the number patterns.

- **Counting the numbers out loud** allows the student to hear the rhythm in the numbers and may help in recognizing the pattern. Often times, number patterns such as 2s, 5s, and 10s, are said out loud and are not always connected with a visual representation. In this case, counting the numbers verbally might assist the student in recognizing the pattern. A teaching strategy would include having the student count out loud while shading in the numbers on a 100s chart.
- Depending on the complexity of the pattern, a student may need to **point to the numbers** to gain a sense of the pattern. This strategy will likely be accompanied by counting out loud. This is another strategy to assist the student in recognizing the connection between the visual pattern and the numeric pattern. You can ask what they notice about the numbers that are shaded.
- A student may **focus on the numbers that are not highlighted**. There are patterns there as well, but the 100s chart may be overwhelming, and it may be difficult to discriminate between the highlighted and not highlighted numbers. Since the goal of the activity is to recognize the patterns in the highlighted numbers, a teaching strategy may be to use a number line from 1-10 to begin with to help the student recognize simple patterns and then gradually increase the amount of numbers presented.
- The student who **identifies the visual pattern** (e.g., "It makes lines that go up and down") is not attending to the numbers. The activity is to tell what is special about the numbers, so a teaching strategy might be to work with smaller number sets, such as 1-10, to help the student focus on the patterns in the number, rather than the patterns on the 100s chart. Using vertical number lines may also alleviate confusion. Also, the student can be helped to relate the visual patterns to the number patterns.

thi	5) Patterns, relationships, and algebraic inking. The student recognizes patterns in mbers and operations.	1.5C: The student is expected to compare and order whole numbers using place value.
M	aterials: Number cards to 99	
Pr	ocedure:	
Student will use number cards to order numbers from "least to greatest" and "greatest to least." Numbers should vary in complexity, in order to discover more about the student's understanding of place value (e.g., 35, 45, 55; 62, 65, 67; 48, 50, 56).		
	ace three number cards in front of student. Vary vays on the left or right.	y the placement of numbers so the smaller number is not
	ace these numbers in order from least to greatest to greatest to greatest to greatest to greatest to greatest to	
Ch	neck Student's Response:	Circle Order:
1.	Numbers,,, Correct Incorrect	Least to Greatest Greatest to Least
2.	Numbers,, Correct Incorrect	Least to Greatest Greatest to Least
3.	Numbers,,, Correct Incorrect	Least to Greatest Greatest to Least
4.	Numbers,, Correct Incorrect	Least to Greatest Greatest to Least
Repeat this task with other numbers as needed.		
No	otes:	

1.5C: The student is expected to compare and	Possible interpretations, iss
order whole numbers using place value.	and implications for teaching

sues to follow up on, ing

This task requires students to use their understanding of place value to compare and order numbers according to their values. Pay attention to the student's errors, if any, and plan further questions and teaching strategies to address these errors.

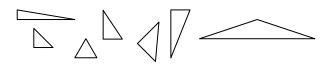
- There may be one place value that consistently produces errors (e.g., student orders numbers according to ones digits and does not consider differences in tens digit). A teaching strategy to address this issue might be to practice ordering numbers using those that vary by tens digit only (e.g., 40, 50, 60). This will help the student pay closer attention to the value of the tens place when ordering numbers. Also, provide practice in counting by 10s.
- The student may have more difficulty ordering the numbers from greatest to least. In this case, the student may order numbers by the difference in the digits in the ones place, rather than the tens place (e.g., 47, 53, 42). A teaching strategy might be to have the student order numbers that only vary by tens (e.g., 41, 51, 71). Providing a change in only one place value limits distracters and allows the student to focus on the problem of "greatest to least" one place value at a time.
- If the student is **having difficulty ordering numbers**, simplify the numbers presented (e.g., changing in only one place value at a time) and provide base ten blocks to assist the student with physical manipulation of the blocks.
- If the student **does not verbalize a strategy**, it is always good to ask how he or she decided on the order. This may provide you with further information regarding the student's understanding of place value – both when the order is correct and incorrect.

(1.6) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

1.6A: The student is expected to describe and identify two-dimensional geometric figures, including circles, triangles, rectangles, and squares (a special type of rectangle).

Materials:

Two-dimensional geometric figures (can be drawn or printed onto index cards, or can be teacher-cut or commercially made from other material). Be sure to include shapes that are non-canonical. For example, include a diverse collection of triangles:



Procedure:

Show the student the following from the collection, one at a time (circle, triangle, square, rectangle).

How can you describe this two-dimensional geometric figure?

Circle shapes student correctly identifies. Put a slash through shapes a student fails to identify.

Circle		Circle, non-canonical, e.g., very small or very large
Triangle	Triangle in atypical position, e.g., base up	Triangle, non-canonical, e.g., with high skew ratio
Rectangle	Rectangle in atypical position, e.g., rotated 45°	Rectangle, non-canonical, e.g., 1:10 length: width ratio
Square-rectangle	Square in atypical position— e.g., rotated 45°	Square, non-canonical, e.g., very small or very large

Check Student's Strategies:

□ Student only identifies prototypical shapes such as equilateral triangle

□ Student only identifies shapes that are in prototypical positions. For example, student identifies this shape as a square □ but insists this shape is not a square: ♦

 \Box Other:

Notes:

1.6A: The student is expected to describe and		
identify two-dimensional geometric figures,		
including circles, triangles, rectangles, and squares		
(a special type of rectangle).		

Possible interpretations, issues to follow up on, and implications for teaching

- If the student **only identifies shapes that are prototypical**, such as equilateral triangles, a teaching strategy would include challenging the student's somewhat rigid conception of why a triangle is a triangle. An activity might involve asking students to tell you rules about triangles. For each 'rule,' draw a figure that violates the 'rule' on a piece of chart paper. For example, if a student suggests that all triangles are pointy, draw a figure that is pointy but is not a triangle.
- If the **student only identifies shapes that are in prototypical positions**, a teaching strategy could include challenging the student's conceptions about position. For example, if a student insists that a square turns into a diamond when rotated, ask questions such as, "How do you know?" and "What changes when we rotate this figure?" "How do you define a square?" etc. You could also ask them to take a triangle and rotate it around, and then ask them, "Is it different, or is it the same?"

(1.6) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

1.6B: The student is expected to describe and identify three-dimensional geometric figures, including spheres, rectangular prisms (including cubes), cylinders, and cones.

Materials:

Three-dimensional geometric figures, including spheres, cubes, rectangular prisms, cylinders, and cones.

Procedure:

Present student with one three-dimensional geometric figure at a time.

What is the name of this three-dimensional geometric figure? How do you know it is a [*figure*]?

Chaok Student's Despenses
Check Student's Response:
Figure
□ Correctly identifies figure
□ Incorrectly identifies figure
□ Identifies attribute(s) of figures: number of vertices number of corners other
□ Identifies non-geometric attributes (e.g., "because it looks like an ice-cream cone")
□ Other:
Figure
□ Correctly identifies figure
□ Incorrectly identifies figure
□ Identifies attribute(s) of figures: number of vertices number of corners other
□ Identifies non-geometric attributes (e.g., "because it looks like an ice-cream cone")
□ Other:
Figure
□ Correctly identifies figure
□ Incorrectly identifies figure
□ Identifies attribute(s) of figures: number of vertices number of corners other
□ Identifies non-geometric attributes (e.g., "because it looks like an ice-cream cone")
□ Other:
Notes:

1.6B: The student is expected to describe and identify three-dimensional geometric figures, including spheres, rectangular prisms (including cubes), cylinders, and cones.

Possible interpretations, issues to follow up on, and implications for teaching

- If the student **identifies the names of the shapes but says little else**, a teaching strategy would include challenging the student to sort shapes based on attributes. This can be done in a number of ways. For example, you might ask each student to create a group of five shapes that have something in common. Students can then guess what their peers' collections have in common. (Your students will probably figure out that this game is more fun when the shapes in their groups are diverse and peers have to work to figure out what they all have in common). Don't be too concerned if students do not yet use formal mathematical language to describe attributes. Categorizing shapes as "really pointy ones" or "ones that are like bridges" is a first step in shape analysis. It is important for students to think critically about attributes that they understand before they can think critically about attributes that are mathematically relevant.
- If the student **describes attributes of shapes informally** (e.g., "there are three sides," and/or "there are three pointy things," etc.) a teaching strategy might include challenging students to express their thoughts regarding shape analysis to their peers. For example, students can be asked to share their sorting rules with the entire class—this will challenge students to move beyond descriptions such as "really pointy ones" when they realize other students may have used the same 'rule' for very different shapes).
- If the student **identifies the figures based on non-geometric attributes**, this still tells you something about what she or he is thinking. In this case, she or he may be more focused on the ways she or he experiences these objects in the real world. And while her or his comparisons may be accurate, the goal of the task is to find out about her or his understanding of geometric attributes. A teaching strategy might be to draw her or his attention to the physical attributes of each shape (e.g., "You're right, you can't roll a cube. Maybe that is because it has flat faces and the sphere has no flat faces. Can you tell me another way these two are different or similar?").

(1.6) Geometry and spatial reasoning. The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.		1.6C: The student is expected to describe and identify two- and three- dimensional geometric figures in order to sort them according to a given attribute, using informal and formal language.
Materials: Teacher-made, stu commercially-mad Cylinder Cone Cube		Triangle with high skew ratio
		Circle Pentagon Hexagon Octagon Other:
Procedure:		
Place shapes on table.		
I'm going to give you some clues about one of these objects, and I want you to use the clues to find the object.		
(1) The first clue is that this shape has only straight sides/lines (pause for 5 seconds). The second		

- (1) The first clue is that this shape has only straight sides/lines (*pause for 5 seconds*). The second clue is that this shape has four sides (*pause for 5 seconds*). All four of the sides are the same size— they are all the same length (*pause for 5 seconds*).
- (2) I'm only going to give you one clue for this one. This object has six faces and all of the faces are the same size.
- (3) There might be more than one match for this next one. These things have no vertices or edges.

 $\stackrel{\frown}{\longrightarrow}$ Try making up your own clues for other figures and have students do the same.

Check Student's Response:	Check Student's Strategies:	
Square	Square	
	□ Counts sides of figures	
□ Other:	Compares shapes with straight sides versus shapes without straight sides	
	\Box Examines lengths of sides on figures with straight sides	
	□ Selects a figure without counting sides or comparing it to any other figures in a way that is observable to you	
	□ Other:	
Cube	Cube	
□ Correct	□ Counts faces of figures	
□ Other:	Compares shapes with straight sides versus shapes without straight sides	
	\Box Examines lengths of sides on figures with straight sides	
	Selects a figure without counting sides or comparing it to any other figures in a way that is observable to you	
	□ Other:	
Circle, Sphere	Circle, Sphere	
□ Circle □ Sphere	 examines vertices of figures (eliminates figures with vertices) 	
\Box Other:	□ Other:	
Notes:		

1.6C: The student is expected to describe and **identify** two- and three- dimensional geometric figures in order to sort them according to a given attribute, using informal and formal language.

Possible interpretations, issues to follow up on, and implications for teaching

Square

- If the student **counted the sides of figures** (to determine which figures have four sides), the student may very well have an understanding of this mathematical vocabulary word and the underlying concept. A teaching strategy could involve challenging the student to think more about sides. An activity might involve asking students to draw figures with a certain number of sides, then discuss how the figures are alike and how they are different.
- If the student **compared figures with straight sides versus figures without straight sides** (to exclude figures without straight sides or include figures with straight sides), the student may very well have an understanding of what straight means. A teaching strategy could involve challenging the student to think more about what it means for something to be straight. An activity might involve asking students to draw figures with straight sides, then discuss how the figures are alike and how they are different.
- If the student **examined lengths of sides on figures with straight sides**, he or she may be ready for measurement activities. Try this activity: Pair students. Give each student instructions individually to create a figure (e.g., a square with sides that are 4 inch cubes long). Students can compare finished figures to determine if they are the same (this involves knowing mathematical language, such as sides, and making precise measurement comparisons).
- If the student **selects a figure without counting sides or comparing it to any other figures** in a way that is observable to you, do you think the student understand the language used in these tasks such as *corner*, *straight*, etc.? If not, do you think she or he understands the concepts underlying such language? One way to assess this might be to ask students to compare figures. Present students with two figures and ask, "How are these the same?" and "How are these different?" This might help you to understand the informal language the student uses to describe such figures. This way you can help them learn more precise mathematical language to describe the figures.

Cube

- If the student **counted the faces of figures** (to determine which figures have six faces), the student may very well have an understanding of this mathematical vocabulary word and the underlying concept. A teaching strategy could involve challenging the student to think more about faces. An activity might involve asking students to draw figures with a certain number of faces, then discuss how the figures are alike and how they are different.
- If he or she **compared shapes with straight edges versus shapes without straight edges**, see above.
- If the student examined lengths of edges on figures with straight edges, see above.
- If the student **selected a figure without counting edges or comparing it to any other figures** in a way that is observable to you, see above.

Circle, Sphere

- If the student **examines vertices of figures and eliminates figures with vertices**, the student may have an understanding of this mathematical vocabulary word and the underlying concept. A teaching strategy could involve challenging the student to think more about vertices. An activity could involve asking students to draw figures with a certain number of sides, then discuss how the figures are alike and how they are different.
- If the student selected figures without attending to vertices, see above.

(1.7) Measurement. The student directly compares the attributes of length, area, weight/mass, capacity, and temperature. The student uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length.	1.7B: The student is expected to compare and order two or more concrete objects according to length (from longest to shortest).		
Materials: Three objects that differ in length (three straws, or blocks, etc.)			
Procedure: Place the objects in front of student in random order.			
(1) Put these in order from shortest to longest .			
(2) Give me the longest straw.			
(3) Give me the shortest straw.			
Check Student's Response:	Check Student's Strategies:		
 1. Ordering blocks/straws according to length Student puts objects in order from shortest to longest Student does not put objects in order from shortest to longest 2. The student gave you: The shorter object The longer object 3. The student gave you: The longer object The longer object The shorter object The shorter object The student gave you: The student gave you: The medium-size object 	 1. Ordering blocks/straws according to length The student aligned the bases of the objects The student did not align the bases of objects 		
Notes:			

1.7B: The student is expected to compare and order two or more concrete objects according to length (from longest to shortest).

Possible interpretations, issues to follow up on, and implications for teaching

1. Ordering blocks/straws according to length

- Did the student **align the bases of the objects**? If so, how precise was he or she in doing this? A teaching strategy might include challenging the student to compare and order the length of objects with very small differences in length (e.g., 29cm versus 29.5 cm). Activities that involve comparing objects with very small differences in length will help the student to appreciate the importance of precision when comparing an attribute such as length.
- If the student **did not align the bases of the objects**, a teaching strategy might involve challenging the student to replicate the length of an object by doing an activity such as cutting a strip of paper to the same length as a given piece of paper. You and the student may find this more useful than traditional teaching practices in which students are simply told to align endpoints/bases of objects to be measured. Telling a student to replicate an empirical procedure such as aligning endpoints of objects does not engage the student in higher-level thought. Challenging the student to cut a strip of paper to the same length as a given piece of paper is more likely to engage the student in higher-level thinking about measurement.

2. Longest

• Did the student give you the longest straw when requested? If not, it might mean that she is unfamiliar with the language used to describe length. A teaching strategy might involve modeling the use of such language and challenging the student to follow suit.

3. Shortest

• Did the student give you the shortest straw when requested? Many children seem to understand words like longer and bigger before they understand words such as shorter? Do you have any idea why that might be? A teaching strategy might involve modeling the use of such language and challenging the student to follow suit.

(1.7) **Measurement.** The student directly compares the attributes of length, area, weight/mass, capacity, and temperature. The student uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length.

1.7D: The student is expected to compare and order the area of two or more two-dimensional surfaces (from covers the most to covers the least).

Materials: 4 pairs of geometric figures cut from cardstock or other sturdy material: <u>Rectangles</u>

- 1 red rectangle, 5cm x 7 cm
- 1 blue rectangle, 6 cm x 7 cm
- <u>Triangles</u>
- Congruent Pair
 - 1 red triangle (base: 10 cm, one side 6 cm, the angle between the base and side: .91 rad)
 - 1 blue triangle congruent (the same size as) triangle above
- Same-Base Pair (heights are different but the bases are the same)
 - 1 red triangle (base 10 cm, one side 7 cm, the angle between the base and side: .91 rad)
 - 1 blue triangle (base 10 cm, one side 6 cm, the angle between the base and side: .91 rad)
- Same-Height Pair (heights are the same but the bases are different)
 - 1 red triangle (base 10 cm, one side 6 cm, the angle between the base and the side: .91 rad)
 - 1 blue triangle (base 11 cm, one side 6 cm, the angle between the base and the side: .91 rad)

Note: the language "congruent" and "same-base" is not to be used with the student.

Procedure:

<u>Rectangles</u>: Hand the student two rectangles (one in each hand).

Here are two rectangles, a red one and a blue one. Are they the same size or different sizes? (Wait for response.)

If the student replies that the figures are different sizes, ask,

Which is bigger (or covers more, or has more area), the red one or the blue one?

<u>Congruent triangles</u>: Hand the student two congruent triangles (one in each hand). Here are two triangles, a red one and a blue one. Are they the same size or different sizes?

(Wait for response.)

If the student replies that the figures are different sizes, ask,

Which is bigger (or covers more, or has more area), the red one or the blue one?

Same-base triangles: Hand the student two same-base triangles (one in each hand).

Here are two triangles, a red one and a blue one. Are they the same size or different sizes? (Wait for response.)

If the student replies that the figures are different sizes, ask,

Which is bigger (or covers more, or has more area), the red one or the blue one?

<u>Same-height triangles</u>: Hand the student two same-height triangles (one in each hand). Here are two triangles, a red one and a blue one. Are they the same size or different sizes? (Wait for response.)

If the student replies that the figures are different sizes, ask,

Which is bigger (or covers more, or has more area), the red one or the blue one?

Check Student's Responses:	С	Check Student's Strategies:
 Rectangles Says the rectangles are the second s	same size igger, covers gger/covers tume size ger, covers more,	 Rectangles Places one figure over another Places figures side-by-side Does not manipulate figures Striangles (Congruent) Places one figure over another Places figures side-by-side Does not manipulate figures
 Triangles (Same-Base) Says the triangles are the sa Says the red triangle is bigg has more area Says the blue triangle is big more, has more area 	ume size ger, covers more,	 riangles (Same-Base) Places one figure over another Places figures side-by-side Does not manipulate figures
 Triangles (Same-Height) Says the triangles are the satisfier size/congruent Says the red triangle is bigg has more area Says the blue triangle is bigg more, has more area 	ume ger, covers more,	 Triangles (Same-Height) Places one figure over another Places figures side-by-side Does not manipulate figures
Notes:		

1.7D: The student is expected to compare and order the area of two or more two-dimensional surfaces (from covers the most to covers the least).

Possible interpretations, issues to follow up on, and implications for teaching

Rectangles

- If the student **placed one figure over another**, he or she used a very efficient strategy. Rather than rely purely on perception or simply looking at the figures in order to make the comparison, he or she applied a direct and sensible method of comparison. In order to further this student's development, a teaching goal might involve challenging this student to make such comparisons indirectly, using transitive reasoning. For example, the student could be challenged to compare the areas of two rectangles on different desks. You might provide tools such as rulers, and tiles and encourage the student to use the tools to determine which rectangle is larger without comparing them directly. Once the indirect comparison has been made, the student should be encouraged to check his or her work by comparing them directly.
- If the student **placed the figures side-by-side**, how do you think he or she came up with her answer? Was the size difference obvious enough that the student didn't need to analyze the figures? If so, a teaching goal might involve challenging the student to make size comparisons by closely analyzing the attributes of figures that differ in size only slightly (e.g., 9:10 ratio). Remember: students this age may find it difficult to direct their attention to length and width at the same time—it is a lot of information to deal with! Be patient. Some students may not grasp this for years.
- If the student **did not manipulate the figures** why do you think that was? See above for ideas. Did the student understand that the attribute of interest was the size/area of the rectangle? If not, a teaching goal might involve providing the student with experiences in comparing attributes in various contexts (e.g., the height of students, the length of blocks, etc.).

Triangles

- If the student **placed one figure over another**, he or she used a very efficient strategy. Rather than rely purely on perception, the student applied a direct and sensible method of comparison. In order to further this student's development, a teaching goal might involve challenging the student to make such comparisons indirectly, using transitive reasoning. For example, the student could be challenged to compare the areas of two triangles on different desks. A student may or may not be aided by a tool such as a ruler in this task. It might be more useful to carefully draw the triangles to be compared on 1cm graph paper. This way, the student can count how many full and partial squares each triangle takes up and make the comparison using numbers (this is also a great introduction to fractions and rational numbers).
- If the student **placed the figures side-by-side**, how do you think he or she came up with her answer? Was the size difference obvious enough that the student didn't need to analyze the figures? If so, a teaching goal might involve challenging the student to make size comparisons by closely analyzing the attributes of figures that differ in size only slightly (e.g., 9:10 ratio). Remember: students this age may find it difficult to direct their attention to length and width at the same time—it is a lot of information to deal with! Be patient—some students may not grasp this for years.
- If the student **did not seem to manipulate the figures**, why do you think that was? It could simply be too much information for the student to handle. A teaching goal might involve providing this student with more experiences comparing attributes such as length in various contexts.

(1.8) Measurement. The student understands that time can be measured. The student uses time to describe and compare situations.	1.8B: The student is expected to read and write time to the hour and half-hour shown using analog and digital clocks.		
Materials: • Analog clock or pictures representing time on analog face, • Pictures of times on digital face Varying times by hour and half hours (e.g., 10:00, 2:30, 5:00, 12:30, etc.)			
Procedure: Present student with analog or digital representations.			
What time does the clock show?			
Check Student's Response:	Check Student's Strategies:		
 1. Time: Analog Digital Correct Hour correct but not minute Confuses hour and minute Incorrect 2. Time: Analog Digital Correct Hour correct but not minute Confuses hour and minute Incorrect 	 Used fingers to point at clock face Said time without pointing or counting Other: Used fingers to point at clock face Said time without pointing or counting Other: 		
 3. Time: Analog Digital Correct Hour correct but not minute Confuses hour and minute Incorrect Notes:	 Used fingers to point at clock face Said time without pointing or counting Other: 		

1.8B: The student is expected to read and write time to the hour and half-hour shown using analog	Possible interpretations, issues to follow up on, and implications for teaching
and digital clocks.	

This activity assesses students' abilities to read and write time. The digital clock will likely present fewer problems for students as they need only to read the numbers. For the analog face, determining time on the half hour may be more difficult. Observe the students' strategies for determining the time.

- For students who **confuse the hours and minutes**, a teaching strategy would include working with analog faces that are only on the hour so the students gain practice corresponding the small hand with the hour and the large hand at the 12. Once they are more comfortable with recognizing what each hand represents, move on to half hours.
- Students who **correctly read and write the times shown** may be ready to work with times in quarter hours or 5-minute increments.