(K.1) **Number, operation, and quantitative reasoning**. The student uses numbers to name quantities.

K.1A: The student is expected to use one-to-one correspondence and language such as more than, same number as, or two less than to describe relative sizes of sets of concrete objects

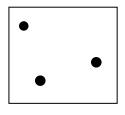
Materials:

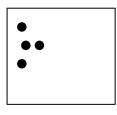
- 3 mats (or paper plates anything that will draw the student's attention to three different areas)
- Approximately 20 objects that can be counted (counting bears, chips, etc.)

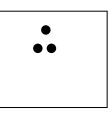
Procedure: When arranging the objects on the mats, do so in a haphazard way: don't arrange items into a formation. Also vary which mat has the larger number: don't fall into a recognizable pattern.

1. Place 3 objects (spread out) on one mat, 4 objects (clustered) on another mat, and 3 objects (clustered) on the third mat.

Which mat has the more [objects]?

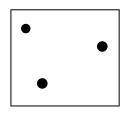


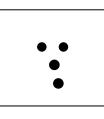


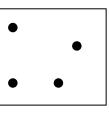


2. Place 3 objects (spread out) on one mat, 4 objects (clustered) on another mat, and 4 objects (spread out) on the third mat.

Which mat has fewer [objects]?

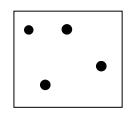


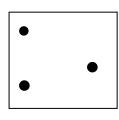


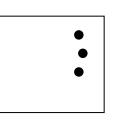


3. Place 4 objects (spread out) on one mat, 3 objects (spread out) on another mat, and 3 objects (clustered) on the third mat.

Which mats have the same number of [objects]?







You could also do this activity with groups of three and five counters.

Check Student's Responses:	Check Student's Strategies:
 The student identified: The mat with more objects The mat on which the objects were most spread out A mat with less objects Other: 	 The student: Counted each object on each mat Just looked, then chose Other:
 2. The student identified: The mat with less objects The mat on which the objects were least spread out A mat with more objects Other: 	 2. The student: Counted each object on each mat Just looked, then chose Other:
 3. The student identified: The mats with the same number of objects Two mats not having the same number Other: 	 3. The student: Counted each object on each mat Just looked, then chose Other:
Notes:	

K.1A: The student is expected to use one-to-one correspondence and language such as more than, same number as, or two less than to describe relative sizes of sets of concrete objects.

Possible interpretations, issues to follow up on, and implications for teaching

- If the student **counted each object on each mat and made comparisons accurately**, he or she may be well on his or her way to comparing sets of objects based on quantity.
- If the student **counted each object on each mat but did not always make comparisons accurately** (the student may have correctly counted each object in each set, but still declared that the set of objects most spread out was the most), you may be interested to learn whether the student understands that the last number counted in a set of objects is equal to the number of objects in the set — an aspect of cardinality. In order to further assess the student's understanding of this concept, you may find it useful to present a set of objects to the student, ask the students to count the objects, then move the objects around again and ask how many they are. You will probably find that many students insist on counting and recounting the objects even though none were added or taken away during the transformations. A teaching strategy would include challenging students to share their reasoning during activities in which they are asked to count a set of objects that are placed in a variety of positions.
- If the student **did not count objects aloud**, how do you think he or she determined the response? Do you think he or she was counting objects? Was he or she making judgments based on the amount of space each set of objects covered, rather than counting each set of objects and making the comparison based on the number of objects in each set? A teaching strategy would involve providing the student with experiences comparing sets of objects that take up different amounts of space.
- If **no strategies were observed**, how do you think the student came up with his or her answers? You can ask the student how he or she decided. Do you think the student understands mathematical language such as more, same, and less? Teaching students about mathematical language such as more, same, and less is very important. Be patient—many children struggle with this. More than and same as are usually understood before less than. Modeling this mathematical language in various contexts and challenging students to do the same is a great teaching strategy.

As follow-up, you could ask the student to create sets with more or less counters. For example, give the student three counters and ask them to create a set with two more counters.

(K.1) **Number, operation, and quantitative reasoning**. The student uses numbers to name quantities.

K.1B: The student is expected to use sets of concrete objects to represent quantities given in verbal or written form (through 20).

Materials: 20 countable objects (counting bears, or similar objects)

Procedure: Place objects on table within reach of student

- (1) Please give me 5 [objects].
- (2) Please give me 9 [objects].
- (3) Please give me 14 [objects].
- (4) Please give me 18 [objects].

◊ Note: Make sure the collection has 20 objects. You can repeat the task with different numbers.

For additional rigor, you could prompt students with a written number, such as "Please give me [show the number 5] objects."

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Check Student's Response:	Check Student's Strategies:
1. The student gave you:	\Box gives you one object at a time
\Box 5 objects	pushes aside/points at objects, then gives you the entire group of objects
□ another number	□ almost instantaneously gives you some or
2. The student gave you:	all of the objects (in what appears to be an arbitrary way)
□ 9 objects	□ other
□ another number	
3. The student gave you:	
\Box 14 objects	
□ another number	
4. The student gave you:	
□ 18 objects	
□ another number	
Notes:	

K.1B: The student is expected to use sets of concrete objects to represent quantities given in verbal or written form (through 20).

Possible interpretations, issues to follow up on, and implications for teaching

- If the student **gives you one object at a time and counted aloud**, did he or she repeat the last number in the counting sequence (e.g., 7...8...8 [objects])? If he or she didn't repeat the last number in the counting sequence, does he or she understand that each counting word corresponds to one object AND the total number of all objects counted up until that point? This is an aspect of cardinality—that the last number indicates how many. Try this activity to investigate this concept more thoroughly with pairs of students: present one student with a given number of objects. Say, "here are six blocks" (feel free to substitute other numbers and materials). Say, "Please give six blocks to [another student]." Keep track of the strategies students use in this exercise. You will probably find that students who at first insisted on counting blocks might find a quicker way to determine the correct number of blocks.
- If the student **pushes aside or points at objects then gives you the entire group of objects**, the student might be well on his or her way to counting out collections of given sizes and recognizing that the number in a set remains the same even if the objects are moved into different formations. If the student seems to have a good grasp of these concepts, you could consider going beyond the grade level expectation and introducing very large numbers along with base-10 blocks (does the student recognize that he or she can count units of ten by tens, or does he or she need to count each unit individually?). While not every student will be ready for moving beyond 20 objects in kindergarten, a student who demonstrates proficiency might be.
- If the student **almost instantaneously gives you some or all of the objects in what appears to be an arbitrary way**, don't fret. Counting out collections of objects involves many skills. In order to be successful at this task, the student needs to be able to know counting words, apply counting words to objects once and only once, and remember how many objects you have requested. A teaching goal might involve challenging the student to produce small collections of objects. Start with just two or three items and gradually increase the number when the student seems ready.

(K.1) Number, operation, and quantitative reasoning . The student uses numbers to name quantities.	K.1C: The student is expected to use numbers to describe how many objects are in a set (through 20) using verbal and symbolic descriptions.	
Materials: 20 countable objects (counting bears, or	chips, or similar materials)	
Procedure: Place some number of objects within reach of the student, in a haphazard arrangement: e.g. 8, 10, 14, 18, 20 *		
How many [objects] are there?		
You could also ask the student to respond with symbolic	olic descriptions by asking them to write the answer.	
* These numbers are just suggestions. Try this task with numbers that you think are appropriate for each student. If a student seems to find this task easy present the task again with more objects and/or arrange the objects in different formations. If the student finds the task to be difficult, try again with a smaller number of objects.		
Check Student's Response:		
Correct Another number:	No response	
Check Student's Strategies:		
□ Counted objects by touching each one once and only once – but did not say numbers out loud.		
□ Touched objects and said counting numbers out loud for each one		
□ Said number words aloud but did not apply them to objects counted (e.g., pointed finger haphazardly at some objects and said counting words aloud that did not necessarily correspond to objects being counted)		
Double counted (e.g. counted one or more obje	ects more than once)	
Moved objects into a different formation (e.g., positioned objects into a line, etc.) then counted each object once and only once		
□ Said counting numbers aloud without touching	objects	
□ Looked at objects without touching them		
\Box None observed		
Notes:		

Possible interpretations, issues for follow up, and implications for instruction	K.1C: The student is expected to use numbers to describe how many objects are in a set (through 20)
-	using verbal and symbolic descriptions

- If the student **looked at the objects without touching them**, how did he or she figure out how many were in the set? Did he or she count silently? Did he or she look at a subset, and immediately recognize that a cluster of 4 objects was 4 objects and count on from there? Did he or she subitize or immediately recognize the number without counting? While subitizing is a useful strategy, the student might need to know the limits of subitizing. It is a good idea to check all answers, especially those that come almost automatically. A teaching strategy might include teaching students ways to check their answers, in this case, by recounting objects one-by-one.
- If the student **touched objects and said counting numbers out loud for each one**, try some simple addition problems. If the student can do this kind of enumeration, he or she is probably ready since most children start out solving simple addition problems by counting all or counting on. Once the student has counted a set of objects, add one or two more and ask, "How many are there now?" Once the student has answered this question, be sure to follow up with "How do you know?" (even if the student's response was incorrect). Providing students at this stage with simple addition tasks is a great teaching strategy since they will reveal even more about understanding of numbers and counting.
- If the student **arranged the objects into a line** (or similar organized arrangement), does he or she understand that the number labels the total number of objects in the set? If you move the objects around a bit, does he or she need to recount, or does he or she seem to understand that the number of objects is always the same, no matter what the positions of the objects? If you are not sure that the student understands this aspect of cardinality—that the number of objects does not change regardless of the physical arrangement of the objects, a teaching strategy might include providing the student with experiences in which varying numbers of objects are counted in different positions. For example, you could put the objects in a circle to see if the student can count them.
- If the student **said the counting numbers aloud**, did he or she repeat the last number in the counting sequence (e. g, 7...8...8 [objects]?). If he or she didn't repeat the last number in the counting sequence does he or she understand that each counting word corresponds to one object AND the total number of all objects counted up until that point? That is, "three" refers to the third object counted and the sum total of all objects counted up until that point. If you are not sure that the student understands this aspect of cardinality—that the last number indicates how many—a possible teaching strategy could involve challenging the student to determine how many objects are in a set after counting. One way to do this might be to practice counting something like stickers pasted onto an index card. Once the student has counted the number of stickers, hide the card and ask, "How many stickers am I hiding?"
- If **no strategies were observed**, how do you think the student came up with his or her answer? Follow up with an additional task or have a conversation with the student about how he or she came up with the answer. A goal for you as a teacher might be to take time to observe your students carefully – does she ever seem to count during block play or when working with puzzle games or reading story books?

(K.2) Number, operation, and quantitative reasoning. The student describes order of events or objects.K.2B: The student is expected to name the ordinal positions in a sequence such as first, second, third, etc.

Materials: Multiple students

Procedure:

When students are in line (e.g., to walk to another classroom, to walk to the playground, etc.), Ask:

- (1) Who is first in line?Or In which position is [the name of the student who is first in line]?
- (2) Who is third in line?Or In which position is [the name of the student who is third in line]?
- (3) Who is second in line?Or In which position is [the name of the student who is second in line]?
- Note: When asking students to identify ordinal positions, avoid going from first to last. Otherwise, it may appear as if the student is able to identify ordinal positions, when, in fact, she has simply learned to say the names of students or ordinal number names in a rote way.

Check Student's Response:	Check Student's Strategies:
1. First	
Correctly identified name of student first in line/identified position	Selected student whose position was first in line
DID NOT identify name of student first	\Box Verbalized position of student first in line
in line/DID NOT identify position correctly	DID NOT respond correctly (e.g., selected tallest student, or best friend, or another student based on an attribute that may seem arbitrary to you but is meaningful to the student)
2. Second	
Correctly identified name of student second in line/identified position	Selected student whose position was first in line
DID NOT identify name of student	\Box Verbalized position of student first in line
second in line/DID NOT identify position correctly	DID NOT respond correctly (e.g., selected tallest student, or best friend, or another student based on an attribute that may seem arbitrary to you but is meaningful to the student)
3. Third	
Correctly identified name of student second in line/identified position	Selected student whose position was first in line
DID NOT identify name of student	\Box Verbalized position of student first in line
second in line/DID NOT identify position correctly	DID NOT respond correctly (e.g., selected tallest student, or best friend, or another student based on an attribute that may seem arbitrary to you but is meaningful to the student)
Repeat this task with other ordinal numbers (e.g., fou	urth, fifth, etc.).
Notes:	

K.2B: The student is expected to name the ordinal positions in a sequence such as first, second, third,	Possible interpretations, issues to follow up on, and implications for teaching
etc.	

- If the student **said the name of the person** who is [first/second/third] in line **is she able to express positions using ordinal numbers**? A teaching strategy might include providing the student with practice in expressing positions using words such as "first, second third," etc.
- If the student said the ordinal number of the person [first/second/third] in line, can she transfer this knowledge to other situations? For example, if everyone in the line turns 180°, can the student identify who is 1st in line (the student who was initially last would now be first)? A teaching goal might be to provide the student with more experiences using ordinal terms to identify the positions of people, objects, and situations.
- If the student **did not respond correctly**, investigate why that might be. It is important to probe further in order to understand what caused the student's error so that you might correct his or her misconception (you can't do that if you don't understand how the student interpreted your question). Did the student select the last person in line instead of the third person in line because he or she has a vague idea that "third" means "far from first"? A teaching strategy could involve practice using comparative terms, drawing the student's attention to when it is appropriate to use each term (e.g., words like taller and shorter might be used to compare height, words like first and second are used to describe positions of people/objects/things and sequences of events).

(K.4) **Number, operation, and quantitative reasoning**. The student models addition (joining) and subtraction (separating).

K.4A: The student is expected to model and create addition problems in real situations with concrete objects

Materials: Approximately 15 countable objects

Procedure: Place countable objects within student's reach.

I'm going to ask you some questions and you can figure out the answer any way you want.

(1) If you have 2 pencils and I give you 1 more, how many pencils will you have?

(2) If there are 3 monsters in the room and 2 more come in, how many will there be?

(3) If you have 5 french fries and your friend gives you 2 more, how many will you have?

(4) If you have 6 cows in the barn and 3 pigs in the field, how many animals are there?

(5) If you have 2 birds and 1 flies away, how many birds do you have left?

(6) If there are 5 monsters in the room and you get rid of 2, how many are left?

(7) If you have 6 apples and eat 2, how many apples do you have left?

(8) If the tree has 8 branches and you cut 5 off, how many branches does the tree have left?

◊ Note: Feel free to substitute numbers and content in the above questions. Also, it probably isn't a good idea to present all of these to a student at once — just pick a few. The questions increase in difficulty.

Check Student's Response:	Check Student's Strategies:
1. 2+1 □ 3 □ another number:	 counted aloud used the objects used her fingers other:
2. 3+2 □ 5 □ another number:	 counted aloud used the objects used her fingers other:
3. 5+2 □ 7 □ another number:	 counted aloud used the objects used her fingers other:

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4. 6+3 □ 9 □ another number:	 counted aloud used the objects used her fingers other:
5. 2-1 □ 1 □ another number:	 counted aloud used the objects used her fingers other:
6. 5-2 □ 3 □ another number:	 counted aloud used the objects used her fingers other:
 7. 6-2 □ 4 □ another number: 	 counted aloud used the objects used her fingers other:
7. 8-5 □ 3 □ another number:	 counted aloud used the objects used her fingers other:
Notes:	

K.4A: The student is expected to model and create addition problems in real situations with concrete	Possible interpretations, issues to follow up on, and implications for teaching
objects	

What did you observe?

- **Did the student count aloud?** This is a reliable strategy. A teaching strategy could be to encourage the student to become more fluent with computations. He or she will be able to apply these skills to the real-world problems.
- **Did the student use his or her fingers?** This is a reliable strategy for small numbers. It is not efficient, particularly for larger numbers, but it is a good start. Do not respond negatively to this student's strategy. It is important that the student's strategy is accepted; otherwise, he or she may attempt to perform computations in a way that he or she does not understand, which could lead to serious problems down the line.
- Did the student use objects/manipulatives? If so, how did he or she use them? Did he or she count from 1? Did he or she count on from the larger number (e.g., for '5+2' she selected 5 blocks and counted five, six, seven). A teaching strategy for this student might be to encourage him or her to become more fluent with basic number combinations. You will probably find that he or she will use the manipulatives at first, but, with lots of practice, will not always rely on them.
- Very rarely, a student will **provide an answer indicating that the sum is smaller than the larger addend**. Thus, in the case of 6 + 3, the student's answer might be 5. Or the student may respond to 6 -2 with the answer 7. In these cases, you have to go back to basics, giving the student simple addition and subtraction problems with manipulatives and helping him or her to count out the answers.

Ask: How do you know?!

After a student solves a problem regardless of accuracy, ask this important question. Be careful not to make students feel like you are judging them or that there are 'wrong' or 'right' answers when asking 'how did you know' questions. In order to further your students' thinking, you must first figure out what they are thinking.

(K.5) Patterns, relationships, and algebraic	The student is expected to identify, extend, and
thinking. The student identifies, extends, and	create patterns of sounds, physical movement, and
creates patterns.	concrete objects

Materials:

- Objects or pictures with which to construct patterns (differently colored chips, beads, counting bears, or similar objects)
- 2 paper cups or other materials with which you can completely cover objects in the pattern (for example, one cup would be capable of covering one element in the pattern)

Procedure: (Note: you can repeat this task with other patterns)

- (1) Watch me. I'm going to make a pattern. Create AB pattern (e.g., blue, yellow) X 4, that is A, B, A, B, A, B, A, B. What kind of pattern do you see?
- (2) Cover two elements in the pattern with cups or other materials. **Which color is missing here?** as you point to each cup.
- (3) How could we extend the pattern?

Check Student's Response:	Check Student's Strategies:
1. The student:	
□ Labels the objects in the pattern (e.g., "blue, yellow, blue, yellow, blue, yellow, blue, yellow")	
□ Identifies the core (e.g., "It always goes blue, yellow.")	
□ Other:	
2. The student:	
□ Correctly identifies one or more hidden elements.	□ Examines the pattern from the beginning and says colors aloud (e.g., blue, yellow,
Does not identify the hidden elements (e.g., tells you what the colors are on either side of	blue, yellow, bluethis one hiding is yellow)
the hidden element)	□ Identifies the hidden element(s) by looking at elements before/after
3. The student:	
 Correctly continues the pattern Makes a mistake: 	 Says the colors of all of the objects from the beginning of the pattern
	Starts somewhere in the middle and says the colors out loud
	Repeats last element of pattern once and continues
Notes:	

K.5: The student is expected to identify, extend,	Possible interpretations, issues to follow up o
and create patterns of sounds, physical movement,	and implications for teaching
and concrete objects	

1. What kind of pattern do you see?

- If the student **points to each element in the pattern and 'reads' the color**, he or she may not understand that the pattern has a repeating core (the alternation of A and B). A teaching strategy would include challenging this student to analyze patterns to discover their cores. Activities might include completing more tasks predicting the identities of hidden elements.
- If the student **informally describes the pattern's core** (e.g., "It always goes blue, yellow."), a teaching strategy would be to challenge this student to analyze even more complex patterns (e.g., AAB patterns, etc.)

2. Predicting hidden elements

- If the student **examines the pattern from the beginning and says colors aloud** (e.g., blue, yellow, blue, yellow, blue...this one hiding is yellow), he or she may not appreciate that the pattern has a repeating core. A teaching strategy might be to expose the student to similar activities and challenge the student to identify the hidden element.
- If the student **identifies the hidden element(s) by looking at elements before/after**, challenge the student to identify hidden elements in even more complex patterns.
- 3. Continuing the pattern
 - If the student **says the colors of the pattern aloud**, that's great. However, just because the student can replicate and 'read' the colors of the pattern it does not mean that he or she has analyzed the pattern to discover the core. As a teaching strategy, continue exposing this student to diverse patterns in different contexts and challenge him or her to discover hidden elements.
 - If the student **repeats the last element of pattern once and continues** (e.g., if the pattern you constructed ends with a yellow element, the student places another yellow element and then a blue element), it may mean that the student has not discovered the core of the pattern. As a teaching strategy, continue exposing this student to diverse patterns in different contexts and challenge him or her to discover hidden elements.

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(K.5) Patterns, relationships, and algebraic thinking . The student identifies, extends, and creates patterns.	K.5: The student is expected to identify, extend, and create patterns of sounds, physical movement , and concrete objects.
Materials: None	
Procedure:	
(1) Create an "AAB" pattern such as "knee-knee- Ask, What comes next?	clap" and repeat it four times.
(2) Now you do just what I did.	
(3) Now you make your own pattern with your bo	dy, and I'll try to repeat it.
Check Student's Response:	Check Student's Strategies:
 1. The student: □ correctly identifies what comes next □ does not correctly identify what comes next: 	 always repeats numerous elements of the pattern when asked what comes next (e.g., when asked, "What comes next?" after "knee-knee-clap," the student repeats, "knee-knee")
 2. The student: □ repeats the pattern correctly □ makes a mistake: 	when asked what comes next, demonstrates and/or verbalizes element that comes next
 3. The student: □ creates a pattern □ does not create a pattern 	
Notes:	1

K.5: The student is expected to identify, extend,	Possible interpret
and create patterns of sounds, physical movement ,	and implications f
and concrete objects.	

Possible interpretations, issues to follow up on, and implications for teaching

- If the student **always repeats numerous elements of the pattern when asked what comes next** (e.g., when asked, "What comes next?" after "knee-knee-clap," the student repeats, "knee-knee-knee"), he or she may not understand that the pattern has a repeating core. A teaching goal might include challenging this student to analyze patterns to discover their cores. Activities might include completing more tasks predicting the identities of hidden elements.
- If the student **demonstrates and/or verbalizes an element that comes next when asked to do so**, a teaching goal might be to challenge this student to analyze even more complex patterns (e.g., AAAB patterns, growing patterns (A, B, A, A, B, B, A, A, A, B, B, B... etc.)

(K.6) Patterns, relationships, and algebraic thinking . The student uses patterns to make predictions.		nt is expected to use patterns to es next, including cause-and- os	
Materials: Objects or pictures with which to const counting bears, or similar objects)	Materials: Objects or pictures with which to construct patterns (differently colored chips, beads, counting bears, or similar objects)		
Procedure: Watch me very closely. I'm going to make a pattern. Within the student's view, construct a pattern (such as ABABABA, e.g., red-green-red-green).			
(1) What comes next? Point to the right of your pattern.	(2) Why?		
Check Student's Responses:			
1. The student said: C	orrect	Other:	
2. The student continued the pattern with: C	orrect	Other:	
3. The student:			
Provided a reasonable explanation (e.g., "I know that the pattern is red-green-red-green, so the next object is green.").			
□ Did NOT provide a reasonable explanation (e.g., "Because.").			
Notes:			

K.6A: The student is expected to use patterns to predict what comes next, including cause-and-effect relationships

Possible interpretations, issues to follow up on, and implications for teaching

1. What comes next?

- If the student correctly **demonstrates and/or verbalizes the element that comes next when asked to do so**, a teaching goal might be to challenge this student to analyze even more complex patterns (e.g., AAAB patterns, growing patterns (A, B, A, A, B, B,, A, A, A, B, B, B... etc.)
- If the student incorrectly **demonstrates and/or verbalizes the element of the pattern when asked what comes next** (e.g., when asked, "What comes next?" after "red-greenred-green," the student says "green") he or she may not understand that the pattern has a repeating core. A teaching goal might include challenging this student to analyze patterns to discover their cores. Activities might include completing more tasks predicting the identities of hidden elements and explicitly naming the repeating core elements.

2. Why?

- If the student provides a **reasonable explanation**, a teaching goal might include challenging this student to analyze and explain even more complex patterns.
- If the student does not provide a reasonable explanation, a teaching goal might include having this student name the repeating core for various patterns, beginning with simple patterns (such as ABAB), and modeling reasonable explanations for what would come next in patterns.

(K.6) Patterns, relationships, and algebraic thinking . The student uses patterns to make predictions.	K.6B: The student is expected to count by ones to 100.
Materials: None	
Procedure: Ask the student:	
Count as high as you can!	
Check Student's Response:	
Counts to without errors.	
Counts to with one error.	
Notes:	

K.6B: The student is expected to count by ones to 100.

Possible interpretations, issues to follow up on, and implications for teaching

What kinds of mistakes did the student make?

Numbers 1-19

• Did the student **skip numbers**? Which numbers did she skip? Was there a pattern? A teaching strategy would include providing the student with more practice counting aloud. The numbers 1 through 12 don't have a lot of rhyme or reason. The numbers from 13 to 19 are even worse: they show the wrong pattern. If "thirteen" were like "twenty three" or forty three" it should be "ten three"! Be patient with students who have difficulty remembering the arbitrary syllables we take for granted as number words from one to 12 or the strange words from 13 to 19. Even if a student often forgets to say twelve, or some other number, encourage him or her to continue counting as high as he or she she can. Students who have trouble remembering such numbers are often quite capable of counting well above the 20s.

Numbers 20 and above

• Did the student **pause at each new decade** and/or **say something like "twenty-eight, twenty-nine, twenty-10"?** If a student seems stuck when she gets to a new decade, scaffold her by reminding her of decade patterns. For example if the student says, "thirty-*niiiiiine*" and seems stuck, say, "ten, twenty, thirty...." This may be all the student needs to remind her that forty comes next. A teaching strategy would involve providing the student with practice counting in decades above the first two. Ask students to begin counting at other numbers than one (e.g., "Today we're going to start counting from 19.").

(K.7) Geometry and spatial reaso student describes the relative position	0	K.7A: The student is expected to describe one object in relation to another using informal language such as over, under, above, and below.	
 Materials: Four small objects (could be animals, different colored counting bears, etc.) A chair (so that items can be placed into the positions described below) 			
Procedure: Place the objects in the following relative positions.			
on top of	next to	below	
under	between	in front of	
behind	over	above	
Where is [object]? Is it [on top of]	or [behind]? Tel	l me each time.	
If the student seems confused, give him or her two choices. For example, for between, if you place a dinosaur between a bear and a person, ask, Is the dinosaur between the bear and the person or is the dinosaur in front of the bear and the person?			
Circle Student's Response: Student correctly identifies the following spatial prepositions:			
on top of	next to	below	
under	between	in front of	
behind	over	above	
Check Student's Strategies:			
□ The student describes relations of objects to each other using informal language			
☐ The student does not describe relations of objects to each other using informal language, and instead says something like, "It's right there!"			
\Box The student describes the relations of the objects incorrectly			
Notes:			

K.7A: The student is expected to describe one	Possible in
object in relation to another using informal	and implic
language such as over, under, above, and below.	

Possible interpretations, issues to follow up on, and implications for teaching

- If the student **describes relations of objects to each other using informal language**, a teaching strategy would include challenging the student to respond to two positions simultaneously. For example, "behind the table, but also under the couch."
- If the student **does not describe relations of objects to each other using informal language**, a teaching strategy could include challenging the student to use informal language to describe the relative positions of objects. One idea is a game such as "Simon Says." Students can take turns practicing commands such as, "Simon says, put your block above your head."

(K.7) Geometry and spatial reaso student describes the relative positi	0	K.7B: The student is expected to place an object in a specified position.	
•		nt colored counting bears, etc.) positions described below.)	
Procedure: Ask student to place object(s) in relation to chair and/or one another as follows:			
Put the [object] the ch	Put the [object] the chair.		
on top of	next to	below	
under	between	in front of	
behind	over	above	
Check Student's Response: Student correctly places the figurine according to the following spatial prepositions (circle):			
on top of	next to	below	
under	between	in front of	
behind	over	above	
Check Student's Strategies:			
□ Student places objects nonverbally			
\Box Student spontaneously describes the positions of objects he places			
□ Student does not place the objects correctly			
Notes:			

K.7B: The student is expected to place an object in	Possible interpretations, issues to follow up on,
a specified position.	and implications for teaching

- If the student **places the objects nonverbally**, a teaching strategy would include challenging the student to use language to describe the relative positions of objects (See K.8.B).
- If the student **spontaneously describes the positions of objects he or she places** (e.g., "The bear is under the table"), a teaching strategy would include challenging the student to place objects along two dimensions simultaneously. For example, ask the student to "put the bear behind the table and in between the dog and the cat."
- If the **student does not place the objects correctly**, a teaching strategy would include placing objects in various positions and having the student name the position. If the student struggles to name the appropriate position, choose two positions (such as "on top of" and "under") at a time to work on with the student until the student has achieved fluency in being able to name the positions.

(K.8) Geometry and spatial reasoning . The student uses attributes to determine how objects are alike and different.	K.8B : The student is expected to compare two objects based on their attributes.	
Materials: Cutouts of two-dimensional geometric figures (It would be best if the some of the figures are non-canonical. For example, the group should include triangles with high skew ratios in addition to more commonplace equilateral triangles.)		
High skew Equilateral Ratio		
Or: Any other objects that can be compared (buttons) students to think of reasons why things are similar an		
Procedure: Select two figures or have the student select two figures	res.	
Tell me about these two-dimensional geometric figures. (Wait for response.) What is different about these two-dimensional geometric figures? What is the same about them?		
Check Student's Response:	Check Student's Strategies:	
Identifies at least one difference between objects	□ Student identifies the names of the shapes/objects but says little else	
 Identifies at least one similarity Other: 	□ Student describes attributes of shapes informally (e.g., "There are three sides," and/or "There are three pointy things," etc.)	
	□ Other:	
Notes:	1	

K.8B: The student is expected to compare two objects based on their attributes.

Possible interpretations, issues to follow up on, and implications for teaching

- If the student **identifies the names of the two-dimensional geometric figures but says little else**, a teaching strategy would include challenging the student to sort objects based on attributes. This can be done in a number of ways. For example, you might ask each student to create a group of five objects that have something in common. Students can then guess what their peers' collections have in common. (Your students will probably figure out that this game is more fun when the shapes in their groups are diverse and peers have to work to figure out what they all have in common.) Don't be too concerned if students do not yet use formal mathematical language to describe attributes. Categorizing two-dimensional geometric figures as "really pointy ones" or "ones that are like bridges" is a first step in analysis. It is important for students to think critically about attributes that they understand before they can think critically about attributes that are mathematically relevant.
- If the student **describes attributes of two-dimensional geometric figures informally** (e.g., "there are three sides," and/or "there are three pointy things," etc.), a teaching strategy might include challenging the student to express her thoughts regarding analysis to her peers. For example, students can be asked to share their sorting rules with the entire class this will challenge students to move beyond descriptions such as "really pointy ones" when they realize other students may have used the same 'rule' for very different objects).

(K.9) **Geometry and spatial reasoning**. The student recognizes attributes of two- and three-dimensional geometric figures.

K.9C: The student is expected to describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle)

Materials: Shapes, which can be drawn or printed onto index cards, or can be teacher-cut or commercially made from other material. Be sure to include shapes that are diverse and non-canonical. For example:



Procedure:

Show the student a 2-dimensional geometric figure from the collection (like a circle, triangle, square, or rectangle).

What shape is this? How can it be described?

Repeat for other shapes.

Check shapes student correctly identifies. Put a slash through shapes a student fails to identify.				
Circle		Circle, non-canonical, e.g., very small or very large		
Triangle	Triangle in atypical position, e.g., base up	Triangle, non-canonical, e.g., with high skew ratio		
Rectangle	Rectangle in atypical position, e.g., rotated 45°	Rectangle, non-canonical, e.g., 1:10 length: width ratio		
Square-rectangle	Square in atypical position, e.g., rotated 45°	Square, non-canonical, e.g., very small or very large		
Check Student's Competencies:				
□ Student only identifies prototypical shapes such as equilateral triangle				
□ Student only identifies shapes that are in prototypical positions. For example, student identifies				
this shape as a square 🔲 but insists this shape is not a square: 🚫				
□ Other:				
Notes:				

K.9C: The student is expected to describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle)

Possible interpretations, issues to follow up on, and implications for teaching

- If the student **only identifies geometric figures that are prototypical**, such as equilateral triangles, a teaching strategy would include challenging the student's somewhat rigid conception of why a triangle is a triangle. An activity would involve asking students to tell you rules about triangles. For each 'rule,' draw a figure that violates the 'rule' on a piece of chart paper. For example, if a student suggests that all triangles are pointy, draw a figure that is pointy, but is not a triangle.
- If the student **only identifies geometric figures that are in prototypical positions**, a teaching strategy might include challenging the student's conceptions about position. For example, if a student insists that a square turns into a diamond when rotated, ask questions such as "How do you know?" "Why isn't it still a square?" and "What changes when we rotate this figure," etc.

(K.10) Measurement . The student directly compares the attributes of length, area, weight/mass, capacity, and/or relative temperature. The student uses comparative language to solve problems and answer questions.	K.10A: The student is expected to compare and order two or three concrete objects according to length (longer/shorter than, or the same).	
Materials: Three objects that differ in length (straws, blocks, etc.)		
Procedure: Place blocks/straws in front of student in random order.		

Put these objects in order from shortest to longest.

Which object is the longest?

Which object is the shortest?

Check Student's Strategies:
1. Ordering blocks/straws according to length
 Student aligned the bases of the objects Student did not align the bases of objects

K.10A: The student is expected to compare and order two or three concrete objects according to length (longer/shorter than, or the same).

Possible interpretations, issues to follow up on, and implications for teaching

- 1. Ordering blocks/straws according to length
 - Did the student **align the bases of the objects**? If so, how precise was he or she in doing this? A teaching strategy would include challenging the student to compare and order the length of objects with very small differences in length (e.g., 29cm versus 29.5 cm). Activities that involve comparing objects with very small differences in length will help the student to appreciate the importance of precision when comparing an attribute such as length.
 - If the student **did not align the bases of the objects**, a teaching strategy would involve challenging the student to replicate the length of an object by doing an activity such as cutting a strip of paper to the same length as a given piece of paper. You and the student may find this more useful than traditional teaching practices in which students are simply told to align endpoints/bases of objects to be measured. Telling a student to replicate an empirical procedure such as aligning endpoints of objects does not engage the student in higher-level thought. Challenging the student to cut a strip of paper to the same length as a given piece of paper is more likely to engage the student in higher-level thinking about measurement.
- 2. Did the student **fail to give you the longest straw when requested**? If so, it might mean that he or she is unfamiliar with the language used to describe length. A teaching strategy might involve modeling the use of such language and challenging the student to follow suit.
- 3. Did the student **give you the shortest straw when requested**? Many children seem to understand words like longer and bigger before they understand words such as shorter and smaller. Do you have any idea why that might be? A teaching strategy might involve modeling the use of such language and challenging the student to follow suit.

(K.10) Measurement . The student directly compares the attributes of length, area, weight/mass, capacity, and/or relative temperature. The student uses comparative language to solve problems and answer questions.	K.10B: The student is expected to compare the areas of two flat surfaces of two-dimensional figures (covers more, covers less, or covers the same).	
Materials		

Materials:

Two pairs of geometric figures cut from cardstock or other sturdy material: Circles

- 1 red circle, diameter 12 cm •
- 1 blue circle, diameter 10 cm •

Rectangles

- 1 red rectangle, 5cm x 7 cm
- 1 blue rectangle, 6 cm x 7 cm •

Procedure:

<u>Circles</u>: Hand the student two circles (one in each hand). Here are two circles, a red one and a blue one. Are they the same size or different sizes? (Wait for response.)

If the student replies that the figures are different sizes, ask, Which is bigger? Which covers more (or has more) area, the red one or the blue one?

Procedure:

<u>Rectangles</u>: Hand the student two rectangles (one in each hand). Here are two rectangles, a red one and a blue one. Are they the same size or different sizes? (Wait for response.) If the student replies that the figures are different sizes, ask, Which is bigger? Which covers more (or has more) area, the red one or the blue one?

Cheo	ek Student's Response:	Check Student's Strategies:
	es The circles are the same size The red circle is bigger/covers more/has more area	Circles □ Places one figure over another □ Places figures side-by-side □ Does not manipulate figures
	The blue circle is bigger/covers more/has more area	
Recta	angles	Rectangles
	The rectangles are the same size	\Box Places one figure over another
	The blue rectangle is bigger/covers more/has more area	Places figures side-by-sideDoes not manipulate figures
	The red rectangle is bigger/covers more/has more area	
Notes:		

K.10B: The student is expected to compare the areas of two flat surfaces of two-dimensional figures (covers more, covers less, or covers the same).

Possible interpretations, issues to follow up on, and implications for teaching

Circles

- If the student **placed one figure over another**, he or she used a very efficient strategy. Rather than rely purely on perception or simply looking at the figures in order to make the comparison, he or she applied logic to the comparison. In order to further this student's development, a teaching strategy would involve challenging this student to make such comparisons with figures other than circles. (Most students will probably find circles the easiest to compare since it is only necessary to look at one dimension to make a reliable comparison, while figures such as rectangles might have the same lengths but different widths, and so require that students carefully analyze both dimensions.)
- If the student **placed the figures side-by-side**, how do you think he or she came up with his or her answer? Was the size difference obvious enough that the student didn't need to analyze the figures? If so, a teaching strategy might involve challenging the student to use logic and make size comparisons by closely analyzing the attributes of figures that differ in size only slightly (e.g., 9:10 ratio).
- If the student **did not manipulate the figures**, why do you think that was? Did the student understand that the attribute of interest was the size/area of the circle? If not, a teaching strategy would involve providing the student with experiences in comparing attributes in various contexts (e.g., the heights of students, the length of blocks, the relative size of cookies, etc.).

Rectangles

- If the student **placed one figure over another**, he or she used a very efficient strategy. Rather than rely purely on perception or simply looking at the figures in order to make the comparison, the student applied logic to the comparison. In order to further this student's development, a teaching strategy might involve challenging this student to make such comparisons indirectly, using transitive reasoning. For example, the student could be challenged to compare the areas of two rectangles on different desks. You might provide tools such as rulers and tiles and encourage the student to use the tools to determine which rectangle is larger without comparing them directly. Once the indirect comparison has been made, the student should be encouraged to check her work by comparing them directly.
- If the student **placed the figures side-by-side**, how do you think he or she came up with his or her answer? Was the size difference obvious enough that the student didn't need to analyze the figures? If so, a teaching strategy might involve challenging the student to use logic and make size comparisons by closely analyzing the attributes of figures that differ in size only slightly (e.g., 9:10 ratio). Remember: students this age may find it difficult to direct their attention to length and width at the same time—it is a lot of information to deal with. Be patient—some students may not grasp this logic for years.
- If the student **did not manipulate the figures,** why do you think that was? Did the student understand that the attribute of interest was the size/area of the rectangle? If not, a teaching strategy might involve providing the student with experiences in comparing attributes in various contexts (e.g., the height of students, the length of blocks, etc.).