

Copyright Language for Maximizing Algebra II Performance

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LITERATURE REVIEW

“Becoming mathematically proficient is necessary and appropriate for all students” (National Research Council, 2001, p.142). Mathematical proficiency is expected of all students enrolled in the public school system in the state of Texas.

What do we know about the students who are not meeting the state standards for mathematical proficiency? As described in Table 1, Table 2, and Table 3, the student groups with the greater percentages of students failing to demonstrate mathematical proficiency include at-risk learners and English language learners (ELL) in the ESL, Limited English Proficient, and Bilingual categories.

Table 1. *Percentages of Grade 9 Students Failing to Meet Panel Recommendation*

	2003	2004	2005	2006
At-Risk	82	79	72	70
Economically Disadvantaged	72	65	58	58
Special Education	86	80	73	74
ESL	89	86	83	81
Limited English Proficient	89	86	82	81
Bilingual	77	65	59	59

From Texas Education Agency, <http://www.tea.state.tx.us/student.assessment/reporting/results/summary/sum06>

Table 2. *Percentages of Grade 10 Students Failing to Meet Panel Recommendation*

	2003	2004	2005	2006
At-Risk	79	77	77	67
Economically Disadvantaged	68	64	57	53
Special Education	85	81	74	72
ESL	83	83	83	77
Limited English Proficient	83	82	82	77
Bilingual	70	58	56	54

From Texas Education Agency, <http://www.tea.state.tx.us/student.assessment/reporting/results/summary/sum06>

Table 3. *Percentages of Grade 11 Students Failing to Meet Panel Recommendation*

	2003	2004	2005	2006
At-Risk	82	55	48	36
Economically Disadvantaged	72	47	42	37
Special Education	88	69	62	54
ESL	86	67	66	58
Limited English Proficient	85	66	65	57
Bilingual	83	41	53	42

From Texas Education Agency, <http://www.tea.state.tx.us/student.assessment/reporting/results/summary/sum06>

Research on the teaching and learning that are occurring in effective mathematics programs is summed up by Hiebert (2003):

One of the most reliable findings from research on teaching and learning is that students learn what they are given opportunities to learn. “Opportunity to learn” is a significant phrase. It means more than just receiving information. Providing an opportunity to learn means setting up the conditions for learning that take into account students’ entry knowledge, the nature and purpose of the activities, the kind of engagement required, and so on. ...Providing an opportunity to learn what is intended means providing the conditions in which students are likely to *engage* in tasks that involve the relevant content. Such engagement might include listening, talking, writing, and reasoning, and a variety of other intellectual processes.

The teaching component of an effective mathematics program relies on data gained from formative and summative assessment opportunities. (p.10)

To meet the needs of students failing to demonstrate mathematical proficiency, research-based strategies and research-based insights provide direction for addressing the needs of students in at-risk situations, English language learners, and other students who have historically struggled with mathematics. These strategies and insights highlight four primary areas of concern: entry knowledge of the student, nature and purpose of classroom activities, student engagement, and assessment designed to inform instruction.

Entry Knowledge

Entry knowledge is the knowledge and understanding with which a student enters a learning context. Such knowledge is influenced by a student’s background, including linguistic and socioeconomic factors (Ball, 1997), as well as prior educational experiences. Teacher awareness of the extent of a student’s entry knowledge plays a pivotal role in providing instruction for at-risk learners and English language learners (Ball, 1997). “Teachers must learn what their students know so as to know how to approach a topic, and they must also probe what students are learning from lessons” (Ball, 1997, p. 732). Teachers learn about their students’ entry knowledge by listening to what students say without rephrasing what they say (Ball, 1997). As teachers probe student understanding through questioning and instructional conversation and reflect upon what they have heard, the teachers gain a better sense of students’ entry knowledge.

Educators can build on entry knowledge through purposeful vocabulary instruction. Marzano (2004) found that the mean scores for students receiving purposeful vocabulary instruction were 0.97 standard deviation greater than the mean scores of students who did not receive purposeful vocabulary instruction. Attributes of such purposeful vocabulary instruction include introducing of new vocabulary through exposure rather than stated definitions; using language-based and imagery-based representations, such as the Verbal and Visual Word Association strategy (Readance, Bean, & Baldwin, 2001) for vocabulary; refining of vocabulary knowledge through multiple exposures to the terminology in contexts; teaching prefixes, roots, and suffixes enhances students’ understanding of words; facilitating student discourse related to the vocabulary being learned; and emphasizing those words that contribute most to content-area learning (Marzano, 2004).

Nature and Purpose of Activities

When considering the nature and purpose of activities to support learning, one must consider the cognitive goals required for the learning to take place and the instructional design issues that impact those goals. The goals for mathematical learning include mathematical fluency and problem solving proficiency. Mathematical proficiency is an integration of and interdependence between five key elements:

- Conceptual understanding,
- Procedural fluency,
- Strategic competence,
- Adaptive reasoning, and
- Productive disposition (National Research Council, 2001).

A student demonstrates conceptual understanding by giving evidence of comprehension of mathematical concepts, operations, and relations. Evidence of procedural fluency includes demonstrated skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Computational fluency, the knowledge of basic facts and efficient and accurate methods for performing mathematical computations, is embedded within procedural fluency (NCTM, 2000). These conceptual and procedural elements support strategic competence, the “ability to formulate, represent, and solve mathematical problems” (National Research Council, 2001, p. 5). These elements are supported by adaptive reasoning, the “capacity for logical thought, reflection, explanation, and justification” (National Research Council, 2001, p.5). Intertwined with these elements is a student’s productive disposition, his or her “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (National Research Council, 2001, p. 5). Evidence or lack of evidence of each of these elements of mathematical fluency independently, and in concert with each other, provides insight into the extent to which a student demonstrates mathematical fluency.

A second cognitive goal addresses problem solving. “In mathematics, student problem-solving achievement increased with instruction that utilized meaningful, contextualized problems, taught students how to prepare and solve problems systematically, and provided social contexts and peer modeling” (Barley et al., 2002, p. 46). Evidence of mathematical proficiency is demonstrated when students move beyond simple computation to problem-solving contexts.

Instructional Design

Student Needs

Careful consideration must be made when planning for instruction so that each student receives opportunities to develop mathematical fluency and problem solving proficiency. The needs of at-risk learners and English language learners must be addressed when creating instructional designs for learning. There is a broad consensus in studies on at-risk populations on five principles that should guide educational efforts to meet the needs of at-risk learners and English language learners (CREDE, 1997):

- Integrate the efforts of teachers and students in the learning process.
- Embed language and the literacy of instruction in all instructional activities.
- Contextualize teaching and curriculum in the entry knowledge and experiences of the students.

- Challenge students toward cognitive complexity.
- Engage students through discourse, especially instructional conversation.

“Work that is carried out collaboratively for a common objective and the discourse that accompanies the process contribute to the highest level of academic achievement” (CREDE, 1997, p. 2). The discourse should include content-specific vocabulary, questions, problem posing, and representations so that students become literate in communicating about instruction and learning. The process of questioning and sharing thought processes and background knowledge establishes the tone of instructional conversations between students and teachers. As teachers listen carefully to the student, they make conjectures about the student’s intended meanings and adjust their responses to assist the student’s efforts to learn. To support such discourse, “[a]t-risk learners require instruction that is cognitively challenging, that is, instruction that requires thinking and analysis, not only rote, repetitive, detail-level drills” (CREDE, 1997, p. 4). The goal is a balance between fluency and problem solving. At-risk learners and English language learners have a need to learn and practice the language of mathematical fluency and problem solving.

Classroom Practice

Classroom practice that provides opportunities to develop mathematical fluency and problem-solving proficiency is cognitively oriented. Such practice helps students improve the depth and clarity of their thinking, become independent learners, and become more proficient in successfully completing complex, rigorous academic tasks (Barley et al., 2002). At-risk learners benefit from practices where “[t]eachers are modeling, explaining, prompting, and discussing combinations of metacognitive and cognitive strategies” (Barley et al., 2002, p. 47). English language learners benefit when teachers model, explain, prompt, and discuss strategies through different modalities; through connections between new concepts, entry knowledge, and prior learning; through student-generated refinements and reflections about their own work; and through individualized experiences (Huling & Beck, 2005).

At-risk learners also benefit when they model, explain, prompt, practice, and discuss their thought processes and reflect upon these thought processes (Barley et al., 2002). English language learners benefit when partnered with peers to enhance opportunities to model, explain, prompt, practice, and discuss. To support the use of appropriate vocabulary and language, the students and teachers should make frequent use of models, mind maps, word walls, and key vocabulary. These experiences aid in the learning of math content and the English language while sustaining active participation in the learning experience (Huling & Beck, 2005).

Instructional conversations address the needs of at-risk and English language learners. As a practice, instructional conversations between teachers and students and between students provide opportunities to share preliminary solutions, receiving feedback on content and process. Research suggests that students benefit from this cognitively-based classroom practice (Barley et al., 2002). Students also benefit from conversations that require students to compare and contrast concepts and procedures, articulating similarities and differences (Huling & Beck, 2005 and Marzano, 2001).

Analysis of short-term gain suggests that when these classroom practices reflecting effective, cognitively-oriented instruction are used, students benefit. Students show interest in content and tasks, choose to engage in the learning process, and successfully perform rigorous academic tasks (Barley et al., 2002).

Curriculum

“A ‘coherent’ curriculum is one that holds together, that makes sense as a whole; and its parts, whatever they are, are unified and connected by that sense of the whole” (Beane, 1995, p. 3). Curriculum that meets the needs of at-risk learners and English language learners considers “the whole” and “the parts” of content and instructional processes in light of these students’ needs. Learning is enhanced when a teacher identifies specifically the parts and the types of knowledge that form the focus of the “whole” that constitutes each lesson, each unit, and each content (Marzano, 2003). English language learners benefit from seeing a “cohesive big-picture of units and lessons within units” (Huling & Beck, 2005).

Marzano (2003) also determined that “learning requires engagement in tasks that are structured or sufficiently similar for effective transfer of knowledge” (p. 109). Barley et al. (2002) found that activity materials that include varied texts and problem types that are relevant to the students contribute positively to the learning of at-risk students. Additional supporting structures for at-risk students include strategies for analyzing and preparing to meet the demands of texts and problems, “how-to” solutions, procedures, aids to comprehension (Barley et al., 2002). The curriculum for at-risk students should use structured problem-solving tasks while assisting students in developing strategies to successfully work through and learn from these tasks.

English language learners also benefit from challenging, age-appropriate, and well-paced tasks that incorporate contextually-based problems and problem solving. Concepts should be presented accurately, logically, and in engaging ways that incorporate multiple representations including concrete representations, semi-concrete representations, and abstract representations (Huling & Beck, 2005). Marzano (2003) highlights that “learning requires multiple exposures to and complex interactions with knowledge” (p. 112). These repeated exposures and interactions should include contexts centering on conceptual, computational, and procedural fluency as well as strategic competence, adaptive reasoning, and productive disposition (National Research Council, 2001). These curriculum experiences that reflect the teaching and learning of mathematics are the product of interactions among the teachers, the students, and the mathematics in an instructional triangle as shown in Figure 1.

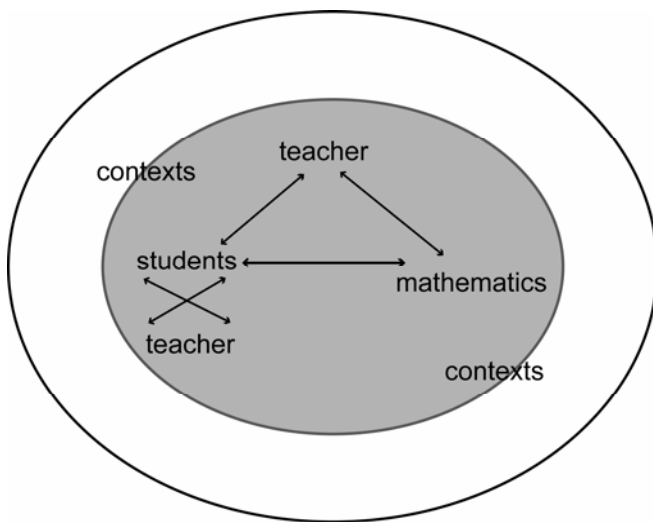


Figure 1. Teaching for Mathematical Proficiency. (National Research Council, 2001, p. 9).

These interactions when well-structured within a curriculum that is focused on appropriate grade-level standards for mathematical proficiency result in the transfer of knowledge described by Marzano (2003). The teaching of such curriculum “promotes learning over time so that the learning yields mathematical proficiency” (National Research Council, 2001, p. 313).

Student Engagement

For at-risk students to engage actively in the content prescribed by mathematics curricula, attention should be given to the developmental, motivational, social, metacognitive, and affective features of instruction (Barley et al., 2002). To address the developmental needs of fifth graders, consideration should be given to the concrete experiences that students need to actively engage in the learning of mathematics content. The National Research Council (2001) found that “[s]imply putting concrete materials on desks or suggesting to students that they might use manipulatives is not enough to guarantee that students will learn appropriate mathematics from them” (p. 353). The manner in which students engage or interact with manipulatives is of tantamount importance.

Students may not look at these objects the same way adults do, and it can be a challenge for students to see mathematical ideas in them. When students use a manipulative, they need to be helped to see its relevant aspects and to link those aspects to appropriate symbolism and mathematical concepts and operations. Observational studies have documented cases in which students were taught to use manipulatives in a prescribed way to perform “wooden algorithms.” If students do not see the connections among object, symbol, language, and idea, using manipulatives becomes just one more thing to learn rather than a process leading to a larger mathematical learning goal (National Research Council, 2001, pp. 353-354).

Manipulatives and other concrete materials provide tools for engaging students at a developmentally appropriate level. However, the planning, activity, and questioning that support the use of manipulatives must be part of “the whole” of the curriculum, part of the thought processes of students, and part of the instructional conversations that take place during mathematics learning.

As students engage with mathematics, they are motivated by feedback on their knowledge gain on conceptual and procedural understandings as well as strategic competence, adaptive reasoning, and productive disposition. Marzano (2003) found that this feedback benefited the learning process.

When individual growth is the criterion for success, then all students can experience success regardless of their comparative status. To accomplish this, two elements are required: (1) an assessment of the achievement level at which students enter a class or unit of instruction, and (2) an assessment of the achievement level at which students exit the class or unit of instruction (Marzano, 2003, p. 149).

Marzano (2003) also found that students are motivated by tasks that are inherently engaging. Covington (1992) and Marzano (2003) suggest that a student's motivation to learn increases when manageable challenges are part of the curriculum and instruction. These activities arouse curiosity, "providing sufficient complexity so that outcomes are not always certain" (Covington, 1992, p. 160). Marzano (2003) found that role playing and instructional games also serve to motivate, and thus engage, students in the learning process.

Motivating activities include a social aspect that promotes student engagement. Intentional planning for social interaction includes planning for cooperative learning. Cooperative learning consists of "students working together in a group small enough that everyone can participate on a collective task that has been clearly assigned" (Cohen, 1994, p. 3). Barley et al. (2002) found that "[c]ooperative learning, when rigorously implemented, can provide [at-risk] students with enriched instruction through peer interaction resulting in improved student achievement" (p. 60).

Rigorous implementation of cooperative learning must address the clarity of directions that are provided to the student. (Repman, 1993 and Barley et al., 2002). Because struggling students tend to be more passive during group learning situations (Repman, 1993 and Barley et al., 2002), research supports arranging for peer groups to generate solutions rather than having individuals generate solutions during the learning process (Barley et al., 2002). Secada and De La Cruz found that

[F]or language minority students in particular, the opportunity to discuss mathematics in a small group may precede competent participation in large group discussion. Studies comparing students' communication in their two languages, in large group discussion and in small groups, have found that language minority students display the lowest level of competency when talking in English during large group discussions, frequently leading to underestimation of children's academic competency (as cited in Brenner, 1998).

Research studies stress the importance of the processes related to mathematical learning that occurs in groups while underscoring the teacher's role as critical to student learning in groups (Barley et al., 2002).

The classroom environment influences the affect of the student, which in turn affects how well the student will engage with his peers and with mathematical learning. When students' affective needs are addressed, student engagement is encouraged. For the English language learners, a learning atmosphere and physical environment that fosters student engagement encourages self-expression and provides positive recognition of students' effort and thinking,

builds student confidence in mathematical proficiency, and fosters an emotionally safe environment that fosters security for thinking and risk-taking for learning (Huling & Beck, 2005). The classroom environment is visually rich, using non-linguistic and linguistic representations to reinforce math-specific vocabulary and concepts (Huling & Beck, 2005; Marzano, 2001). The physical room arrangement facilitates student interaction and group work (Huling & Beck, 2005). Such interactions in turn influence instructional conversation. Instructional conversation improves mathematical proficiency.

Conclusion

To engage students in the learning process, the teacher should use problems and relevant topics that are inviting to the student. The teacher should use instructional conversations. These efforts allow the teacher to assess a student's entry knowledge given the concept or process to be studied.

Complex, rigorous academic tasks provide opportunities for students to develop conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. While completing these tasks, students should engage in instructional conversation with their peers and with their teacher. These instructional conversations offer opportunities for the students to explore mathematical activity at a deeper level while providing insight for the teacher and the students into the students' level of understanding. As students work on academic tasks, collaborative efforts in small groups provide safe environments for students to learn. Students have the opportunity to explore and communicate about the mathematics being learned in a smaller setting that leads to greater comfort in discussing the mathematics in a whole-group setting.

Planning for engagement and tasks should be purposeful. This planning should reflect a coherent sequence for learning, with well-structured tasks to assist students in developing mathematical understanding and understanding of academic processes. The teacher supports students in these tasks by tending to entry knowledge and building background knowledge by addressing mathematical vocabulary. Teachers model, explain, prompt, and discuss thought processes about mathematical content and critical thinking. As they do this, teachers should provide opportunities for students to refine and reflect upon their conceptual and procedural understandings. As at-risk learners and English language learners refine their understanding, they extend their understandings. They also make connections between their new understanding and their entry knowledge.

Teachers should assess student understanding at formative and summative points during instruction. As the teacher assesses student understanding, he or she should provide feedback to the student about growth. The teacher should also use knowledge gained during the assessment process to monitor the effectiveness of instruction, adjusting as needed to increase student success.

PROJECT DESCRIPTION

Overview

The *Maximizing Algebra II Performance (MAP)* Institute is a research-based professional development opportunity for Algebra II mathematics educators. The primary focus of *MAP* is to educate teachers about the inherent alignment between concepts and their connections to other concepts or big ideas vertically with a grade band as well as other strands within the grade level.

MAP utilizes the 5E instructional model, an inquiry-based model of instruction. The 5E lesson structure offers well-timed opportunities to incorporate instructional strategies, such as cooperative learning, vocabulary development, and questioning techniques, that have been proven to impact student achievement. The 5E instructional model encourages a consistent structure for learning with characteristic activities during each phase, so that students can monitor the learning process and gain metacognitive knowledge. The 5E instructional model supports the structure of learning as described by Marzano (2003). Included in the professional development materials are student-ready lessons for each grade level addressed. All student lessons are written using the 5E instructional model. During these lessons, students are engaged in an intensive, rich mathematical experience at the level of cognitive rigor that is demanded by TAKS while addressing the needs of at-risk learners and English language learners.

Instructional Framework

The framework upon *MAP* is built is the 5E instructional model. This model provides the best structure to support the research-based instructional strategies that positively impact a student's mathematical proficiency.

- **ENGAGE:** The instructor initiates this phase by asking well-chosen questions, by presenting a problem to be solved, or by showing something intriguing. The activity should be designed to interest students in the problem and to make connections between past and present learning. The Engage phase of the instructional model facilitates building common ground for all students.

Haynes (n.d.) and Jarrett (1999) propose that English Language Learners (ELL) at the Beginning and Intermediate levels of language proficiency still rely heavily on prior knowledge. They also state that ELL students determined to be at the Advanced and Advanced High levels of English acquisition also benefit from connections to prior knowledge and setting the stage for learning. The Engage phase benefits these students by connecting prior knowledge from past learning to the posed questions, problem, or engaging activity.

- **EXPLORE:** The exploration phase provides the opportunity for students to become directly involved with the key concepts of the lesson through guided exploration that requires them to probe, inquire, and question. As we learn, the puzzle pieces (ideas and concepts necessary to solve the problem) begin to fit together or have to be broken down and reconstructed several times. In this phase, instructors observe and listen to students as they interact with each other and the activity. Instructors provide probing

questions to help students clarify their understanding of major concepts and redirect the questions when necessary.

Jarrett (1999) and Haynes (n.d.) also agree that providing opportunities to explore using concrete models, visuals, patterning, as well as mathematical representations accelerates learning and increases retention for ELL students. The opportunities found in the Explore phase allow instructors to observe and listen to ELL students to determine misconceptions that are language-based and misconceptions that are mathematics-based.

- **EXPLAIN:** In the explanation phase, collaborative learning teams begin to logically sequence events and facts from the investigation and communicate these findings to each other and the instructor. The instructor, acting in a facilitation role, uses this phase to offer further explanation and provide additional meaning or information, such as formalizing correct terminology. Giving labels or correct terminology is far more meaningful and helpful in retention if it is done after the learner has had a direct experience. The explanation phase is used to record the learner's development and grasp of the key ideas and concepts of the lesson.

Barton and Heidema (2002) emphasized the importance of moving ELL from basic interpersonal communication skills to cognitive academic language proficiency. Mathematics is a language in and of itself. The Explain phase of the instructional model provides structure for facilitating the transition from interpersonal communication to mathematical academic language.

- **ELABORATE:** The elaboration phase allows for students to extend and expand what they have learned in the first three phases and connect this knowledge with their prior learning to create understanding. It is critical that instructors verify students' understanding during this phase.

The National Council of Teachers of Mathematics states that all students should have equitable and optimal opportunities to learn challenging mathematics free from racial, gender, socioeconomic status, or language bias. Secada and De La Cruz (1996) found that extending student's knowledge from prior exploration provided optimal instructional opportunities for students to acquire academic proficiency. The Elaborate phase provides these additional learning opportunities.

- **EVALUATE:** Throughout the learning experience, the ongoing process of evaluation allows the instructor to determine whether the learner has reached the desired level of understanding the key ideas and concepts. More formal evaluation can be conducted at this phase (Bybee, 1997).

The 5E instructional model for an inquiry-based lesson fosters strategies that have been shown to impact student achievement. For example, *questioning strategies* are embedded in each phase of the lesson through "Facilitation Questions" provided for the teacher. The teacher poses these questions to students who are struggling with the lesson to guide their

thinking. These questions are designed to prompt independent student thinking so that students may engage in instructional conversations about what they already know about the concepts and procedures, about what they are learning, and about their progress toward mathematical proficiency (CREDE, 1997). *Cooperative learning strategies* are also embedded in the lessons whenever enriched instruction through peer interaction is needed.

Marzano, Pickering, and Pollock (2001) in their book, *Classroom Instruction that Works*, identify nine categories of strategies that have shown to have an effect on student achievement. The nine categories are:

- Identifying similarities and differences
- Summarizing and note taking
- Reinforcing effort and providing recognition
- Homework and practice
- Nonlinguistic representations
- Cooperative learning
- Setting objectives and providing feedback
- Generating and testing hypotheses
- Questions, cues, and advance organizers

The authors suggest three phases educators might include in an instructional unit to utilize the nine categories of strategies. Instructors should begin a unit of instruction by utilizing strategies and well designed questions to provide students an opportunity to connect prior experiences with present learning. The instructor continues to directly involve the students with key concepts of the lesson through guided exploration. The first phase of Marzano, Pickering, and Pollock's instructional planning parallel the Engage and Explore phase of the 5E Instructional model. Students connect prior experiences through well designed questions that focus student's attention and directly involved the student in the learning process.

The second phase included in *Classroom Instruction that Works* is considered to occur during the unit. New knowledge is introduced, students are provided an opportunity to apply new knowledge gained in the unit, and instructors monitor students' attainment of learning goals. Cooperative learning, guided discovery, or any strategies included in Marzano, Pickering, and Pollock's nine categories are encouraged in the lesson design. The 5E instructional model provides a well defined structure for educators to ensure research-based strategies are deployed in a well designed and fluid process to maximize student achievement. Marzano, Pickering, and Pollock's second phase parallel the Explore, Explain, and Elaborate phase of the 5E instructional model.

The third phase of Marzano, Pickering, and Pollock's planning model is defined as the end of the unit which parallels the Evaluate phase of the 5E model. Although instructors are constantly monitoring the progress of their students, the end of the unit serves as the formal evaluation of student learning.

Any one strategy will not effectively work with all students all the time. The model facilitates strategic planning by allowing educators to redefine a lesson as a learning cycle that include a variety of researched-based instructional strategies. The 5E Instructional model for lesson

design provides the structure that naturally incorporates researched-based strategies. Good teaching is good teaching for all students.

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Implementation Strategies

MAP: Considering the Needs of Different Districts and Audiences

Working with Limited Days

Number of Days	Possible Sequence
1	<ul style="list-style-type: none"> • Engage (45 minutes) • Explore/Explain/ Elaborate Some combination of two depending on scope and sequence needs of audience (4.5 hours). • Evaluate (45 minutes)
2	As outlined in presenter materials
3	<p>Day 1:</p> <ul style="list-style-type: none"> • Engage (45 minutes) • Explore/Explain/Elaborate 1 (2.5 hours) • Explore/Explain/Elaborate 2 (2.5 hours) • Homework: Implement lesson. Bring back student work. (15 minutes) <p>Day 2:</p> <ul style="list-style-type: none"> • Analyze student work (45 minutes) • Explore/Explain/Elaborate 3 (2.5 hours) • Explore/Explain/Elaborate 4 (2.5 hours) • Homework: Implement lesson. Bring back student work. (15 minutes) <p>Day 3:</p> <ul style="list-style-type: none"> • Analyze student work (1 hour) • Evaluate – (1 hour) • Apply processing model and 5E instructional model to upcoming unit of instruction (4 hours)

Working with Limited Hours

Number of Hours	Possible Sequence
1	Student lesson appropriate to audience
2	Student lesson appropriate to audience
3	Two student lessons appropriate to audience
4	Explore/Explain/Elaborate 1, 2, 3, or 4

Master Materials List

(for a group of approximately 40 participants)

Sticky notes – 20 pads
Rulers – 40
Highlighters – 40
Tape – 10 rolls
Flip chart markers – 10 sets
Transparency markers
Transparencies – 1 box
Scissors – 10 pair
Tape measures (metric) – 10
Masking tape – 1 roll
Pencils – 40
Chart paper
Stopwatches – 10
Graphing calculators – 40
Meter sticks – 10
String – 1 ball
CBR – 10
Linking Cables – 10
Water bottles (500ml) – 10
Rubber bands
Linguine (1 package)
Film canisters (empty) – 10
Pennies – approximately 50
Patty paper
Teach Timer™

