Leader Notes: Watching the Beat Go On

Purpose:

The purpose of this section of the professional development is to investigate stumbling blocks and challenges that students and teachers have in developing concepts and procedures involved with learning about square root functions.

Descriptor:

The Explore phase has seven parts. The first part sets the stage by introducing the metronome as an instrument to keep musical time. In the second part participants will collect data during a simple simulation of a metronome using a “weight” and “spring” (plastic bottle of water and rubber bands) by measuring the period of the “ticks.” In part 3 participants will analyze the data to establish the inverse relationship between square root and quadratic functions. Then participants will look at transformations to the square root function $f(x)=\sqrt{x}$ using transformation form ($y=a\sqrt{(x-h)}+k$). Part 5 is an extension in which participants may investigate transformations as a result of changes in the coefficient of *x*. In part 6 participants will connect the changing roles of *a*, *h,* and *k* in square root and quadratic functions. Finally, participants will explore several methods for solving square root equations and inequalities in literal and contextual situations.

Duration:

2 hours

TEKS:

a5 Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

a6 Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

2A.9 **Quadratic and square root functions.** The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

2A.9A The student is expected to use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges.

2A.9B The student is expected to relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions.

2A.9C The student is expected to determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities.

2A.9D The student is expected to determine solutions of square root equations using graphs, tables, and algebraic methods.

2A.9E The student is expected to determine solutions of square root inequalities using graphs and tables.

2A.9F The student is expected to analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems.

2A.9G The student is expected to connect inverses of square root functions with quadratic functions.

TAKS™ Objectives Supported:

While the Algebra II TEKS are not tested on TAKS, the concepts addressed in this lesson reinforce the understanding of the following objectives.

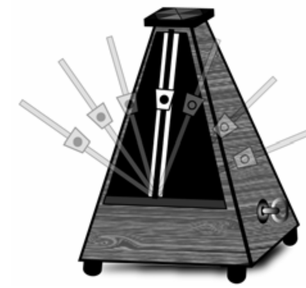
- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 5: Quadratic and Other Nonlinear Functions
- Objective 10: Mathematical Processes and Mathematical Tools

Materials:

Prepare in Advance:	Copies of participant pages
Presenter Materials:	Overhead graphing calculator, metronome or metronome video
Per group:	Filled plastic 500mL bottle, rubber bands (enough to loop together to make a "spring" about 1 meter long), metric tape measure, stopwatch, cards for **Transformations of Square Root Functions Card Sort (**cut apart and placed in plastic bags)
Per participant:	Copy of participant pages, graphing calculator

Explore

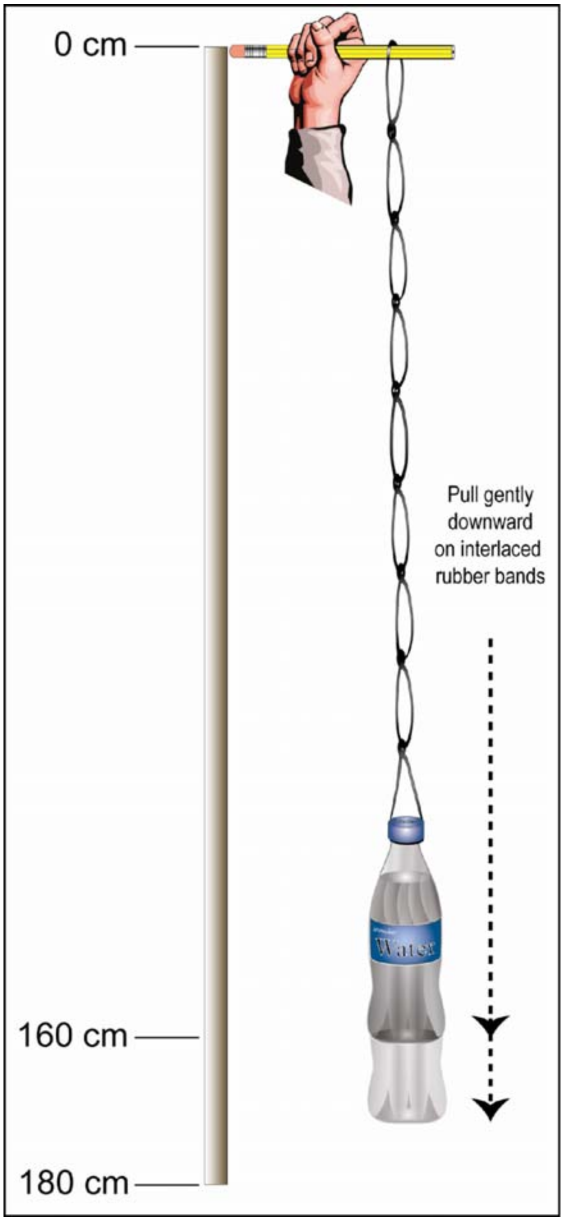
<u>Part 1</u>: Setting the Stage



Leader Notes: If you have access to a metronome, you might want to demonstrate how it works. If you do not have a metronome, a video clip of a pianist playing a piece of music at several speeds with a metronome is included on the presenter CD.

A metronome frequently is used in music to mark exact time using a repeated tick. The frequency of the ticks varies in musical terms from slow *(largo,* about 40-60 beats per minute) to fast *(presto,* 168-208 beats per minute). Individual instrumentalists, choirs, bands, and orchestras all use metronomes to ensure that the beat of the music is consistent with the instructions of the composer and does not unintentionally speed up or slow down while the piece is being played.

What do you notice about the frequency of the ticks and setting of the weight as the music is being played?
The closer the weight is to the base, the faster the ticks and the faster the beat of the music. Conversely, the farther the weight is from the base, the less frequent the intervals and the slower the beats of the music.



<u>Part 2</u>: Modeling and Gathering Data

In this phase of the professional development participants investigate the square root function generated by modeling the period of 10 metronome ticks. In this simplified experiment, use a filled 500 mL bottle suspended from a pencil by a 1-meter strand of rubber bands. (This may vary depending on the elasticity of the rubber bands. When the bottle is attached, the stretched rubber band spring should extend so the bottom of the bottle of water is at least 160 cm below the pencil.) The pencil is held at a height of 160 to 200 centimeters above the floor by a participant, eraser end of the pencil held firmly against the wall so the pencil will not move. The rubber band spring should be attached close to the opposite end of the pencil so the bottle can bounce without striking the wall. To simulate moving the weight up or down on the metronome, the rubber band spring will be wrapped around the pencil until the bottom of the bottle is raised to the desired height. All participants should use the same size bottle, filled to the same height, and the same size rubber bands so the data will be as consistent as possible.

In this investigation, a metric tape measure should be taped to the wall. Participants will suspend a filled 500 mL bottle by a 1-meter rubber band spring from a pencil firmly held perpendicular to a wall at a height of 180-200 centimeters. The difference between the length of the rubber band spring and the suspension height should allow the bottle to bounce without touching the floor.

The length of the rubber band spring will be shortened by wrapping the spring around the pencil so that data is collected at approximately 20-centimeter intervals.

Divide participants into groups of 4. Each person in the group has a job.

Materials manager: Gets the necessary materials, directs the team in setting up the investigation, holds the pencil with the suspended bottle, and shortens the spring when needed.

Measures manager: Measures the distance from the pencil to the bottom of the bottle for each length, initiates the bounce by pulling the suspended bottle down an additional 10 centimeters and counts the bounces (10 at each height).

Time manager: Uses a stop watch to determine the length of each 10-bounce period of time. The time starts when the bottle is released by the measures manager and ends when the bottle completes its 10^{th} bounce.

Data manager: Records the necessary measurements in the table and shares the data with the team.

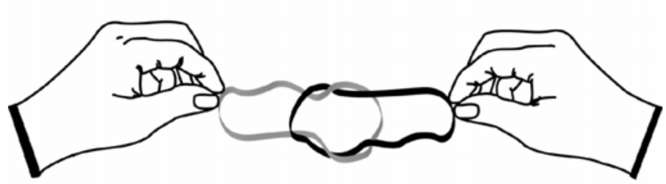
Set-up Instructions

Step 1. **The materials manager should get the necessary materials and ask 2 of the team members to secure the tape measure against the wall. The tape measure should be positioned perpendicular to the floor so that the "zero end" is at 180-200 centimeters above the floor.**

Step 2. **While the tape measure is being positioned, the materials manager and remaining team member(s) build the rubber band spring by looping rubber bands together until the length of the spring is about 1 meter. This task will go more quickly if each person makes part of the spring. Then the pieces can be joined.**

Stretch bands between thumb, forefinger and middle finger.

Merge bands by grasping left band with right forefinger pulling it toward the center.



Pull both bands outward to form an intertwined loop.

Step 3. **Secure one end of the rubber band spring to the pencil and the other around the neck of the bottle. It also works to remove the cap, insert the end of the spring in the bottle, and screw the cap back on.**

Step 4. **The measures manager secures the spring so that it is approximately 160 centimeters in length. After the bottle remains motionless for a few seconds, he should measure the actual length of the spring. The length of the spring includes the length of the rubber band and the length of the bottle.**

Step 5. **The measures manager then pulls the bottle downward about 10 cm and releases it.**

Step 6. **The time manager starts the stopwatch when it is released and stops it at the end of 10 complete bounces. It may be helpful to have all team members count aloud together.**

Step 7. **The data manager records the number of seconds in the table under Trial 1 for 160 cm.** ***Hint: It is more meaningful to start with the spring fully extended and to shorten the spring than to begin at the top and work down.***

Step 8. **Repeat for Trials 2 and 3. Average the data from the 3 trials and record in the Average Time column.**

Step 9. **The materials manager who is holding the pencil shortens the spring by wrapping it around the pencil until the desired length of 140 is obtained.**

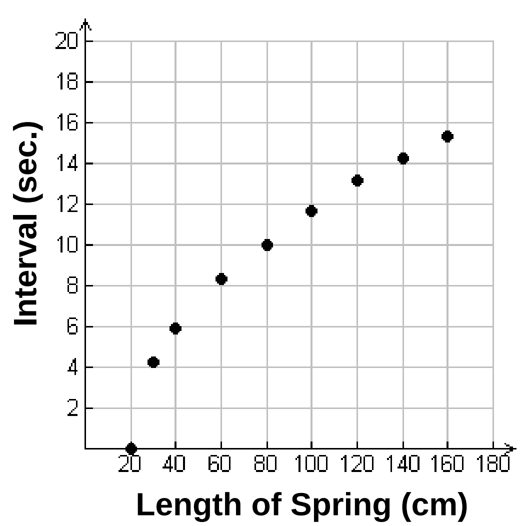
Step 10. **Continue repeating the procedure with shortened lengths of rubber band spring. Continue to record your data.**

1. Record your data in the table below.

Sample data using a bottle 20 cm tall:

Approximate Length of Spring (cm) *x*	Actual Length of Spring (cm) *x*	Trial 1	Trial 2	Trial 3	Average Time (sec) *y*
0	*0*				*Cannot be done*
20	*20*				*0*
30	*30*				*4.23*
40	*40*				*5.92*
60	*60*				*8.30*
80	*80*				*10.04*
100	*100*				*11.70*
120	*120*				*13.17*
140	*140*				*14.22*
160	*160*				*15.34*

2. Make a scatterplot of the data you collected.



3. **What is the independent variable?**
   the length of the spring in centimeters

4. **What is the dependent variable?**
   the period of 10 bounces measured in seconds

5. **Write a dependency statement relating the two variables.**
   The interval of a bounce depends on the length of the rubber band spring.

6. **What is a reasonable domain for the set of data?**
   Responses may vary. The length of the extended rubber band spring with the bottle is approximately 160 cm.

7. **What is a reasonable range for the set of data?**
   Responses may vary. The size and elasticity of the rubber bands used will affect the data. The maximum 10-bounce period appears to be about 16 seconds for this set of data.

8. **Is this data set continuous or discrete? Why?**
   The data set is discontinuous; however, the data points would be closer if we were to shorten the spring by smaller increments. The data theoretically could be continuous, but it is discontinuous in any practical collection of data.

9. **Does the set of data represent a function? Why?**
   Yes, the data set represents a function. For each increase in the length of the spring, there is an increase in the 10-bounce interval.

10. **Write a summary statement about what happened in this data investigation.**
    The shorter the spring, the shorter the interval appears to be. The interval time increases rapidly between intervals when the spring is at short lengths but increases more slowly as the length of the spring increases.

11. **Is the function increasing or decreasing?**
    Increasing

12. **Is the rate of change constant?**
    No, the function values increase rapidly at first and then more slowly.

13. **Does the data you collected appear to be a linear, quadratic, exponential, or some other type of parent function? Why?**
    The data does not appear to be linear because it is not constant. It does not appear to be quadratic because of its behavior in relation to the y-axis, and it seems to lack a symmetric "half." It does not appear to be exponential because its rate of change slows instead of continuing to increase by a constant factor. Based on our explorations in Explore/Explain/Elaborate 1, it might be a square root function because it appears to be the inverse of a quadratic function.

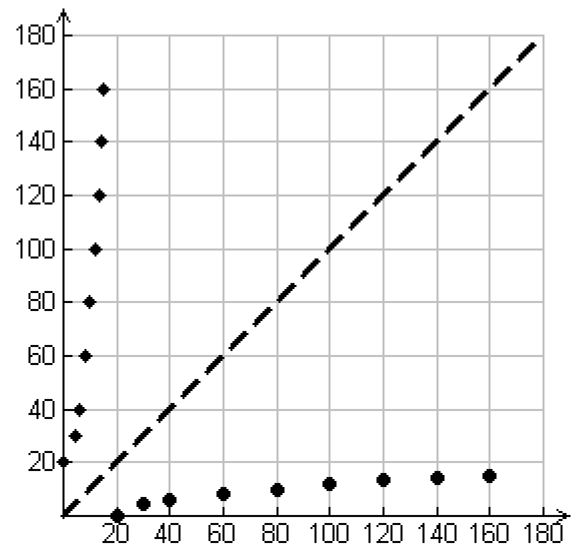
14. How is the bottle bounce activity similar to the ticks of a metronome?
The intervals of the bottle bounce become longer as the spring becomes longer. The intervals of the ticks of the metronome become longer as the length of the rod below the weight becomes longer.

15. What kind of function do you think models the ticking of a metronome? Why?
Probably a square root function because the shorter the rod, the more frequent the tick of the metronomes, just as the shorter the rubber band spring, the more frequent the bounces of the bottle.

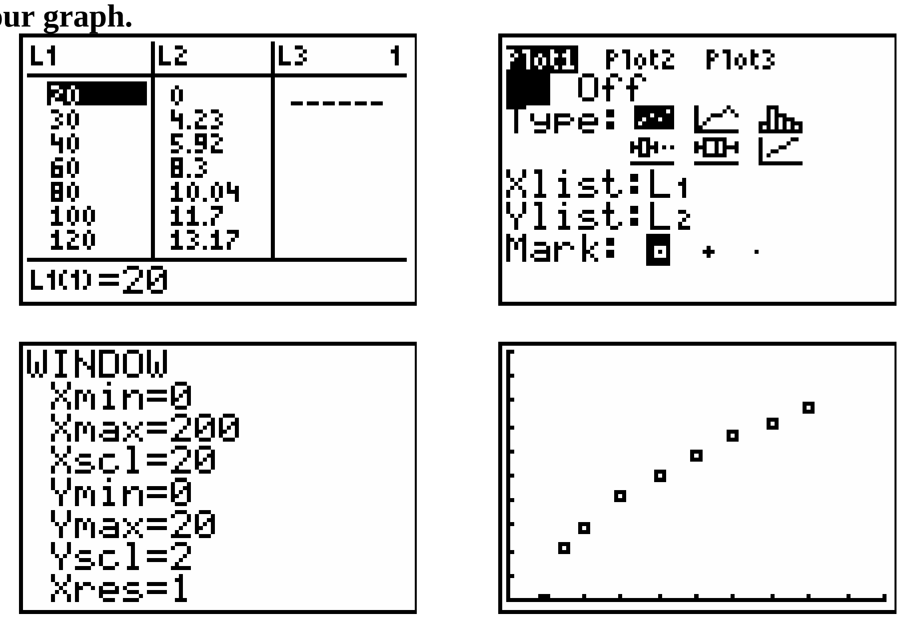
<u>Part 3</u>: Analyzing the Data

1. **How could you determine whether this function is the inverse of another parent function?**
   Answers may vary. The data could be reflected over the line y = x. That is, the x values would become the y values, and vice versa. Or, the L_2 values could be mapped to L_1 using the table feature of the graphing calculator.

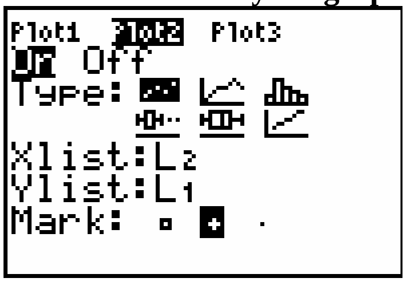
Graphing data from Part 2 and inverse of data simultaneously.

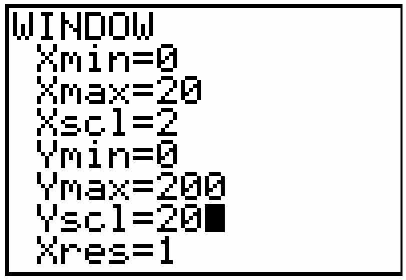


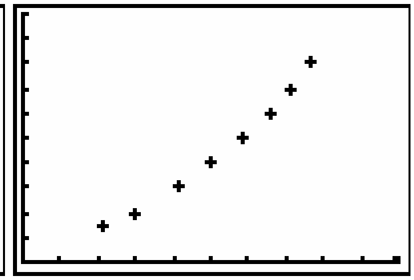
2. **Input your values into L_1 and L_2 of a graphing calculator, letting L_1 be independent values and L_2 dependent values, and create a scatterplot of the original data. Sketch your graph.**



3. Create a second scatterplot that represents an inverse of the data. Use a different plot symbol for this scatterplot. Determine a new domain and range, and set a new viewing window. Sketch your graph.







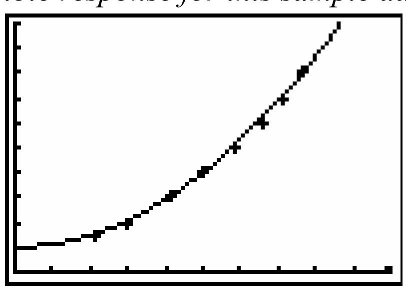
4. What changes must you make to the window to view the second set of data?
reverse the x and y values

5. Which parent function do the *reflected* points most closely appear to represent?
quadratic

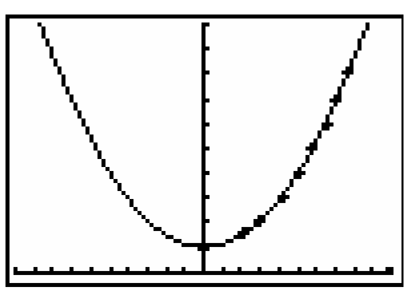
6. How did you determine your function?
Answers may vary.

7. How might you confirm your conjecture?
Answers may vary. Sample response: By checking values in the table.

8. Without using regression, find a function that approximates your data for Plot 2.
Answers may vary. A possible response for this sample data is $y = 0.6x^2 + 20$.



9. Does your viewing window allow you to see both sides of the parabola? If not, readjust your viewing window.
Possible viewing window:

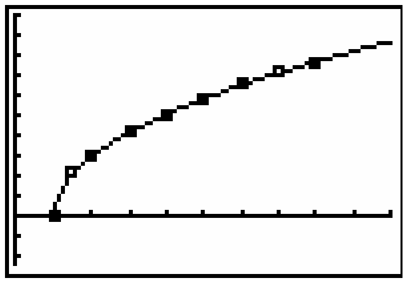


10. How could you use this function to find a function that would approximate the first scatterplot you graphed?

I could find the inverse of the function; in this case, $y = \sqrt{\dfrac{(x-20)}{0.6}}$.

11. Reset your window to view Plot 1. Enter the equation you found in question 8 in the equation editor. Is your graph a close fit to the data in Plot 1?

Possible viewing window representing this sample data



12. Compare and contrast the graphs of a quadratic function and a square root function. How are the graphs similar, and how are they different?

The square root function and the positive side of the quadratic function are inverses. There is no "negative" side to the square root function.

13. Why are there no negative coordinates in the square root function?

Possible responses: If the "half" of the quadratic function were graphed, it would not be a function. It would fail the vertical line test. For this real-life situation, we can have only real numbers as our values.

14. What is the domain of this square root function?

$x \geq 0$

15. What is the range of this square root function?

$y \geq 0$

16. What conclusions can you make about the attributes of a square root function?

Answers may vary but should indicate an understanding of the characteristics of the square root parent function, such as limitations of domain and range.

17. What conclusions can you make about the collected data?

Answers may vary. It can be represented by a square root function.

<u>Part 4</u>: Making Symbolic Generalizations

Distribute Activity Sheet A to each participant and one baggie of equations. Pairs will work together to sort the equations into the proper row and column.

Transformations of Square Root Functions Card Sort

1. Place the cards in the proper row and column.

Description	Example	Example	Notation
Vertical Translation Up	$y=\sqrt{x}+5$	$y=\sqrt{x}+1$	$y=\sqrt{x}+k$ $k>0$
Vertical Translation Down	$y=\sqrt{x}+(-3)$	$y=\sqrt{x}-5$	$y=\sqrt{x}+k$ $k<0$
Horizontal Translation Left	$y=\sqrt{x-(-4)}$	$y=\sqrt{x+5}$	$y=\sqrt{x-h}$ $h<0$
Horizontal Translation Right	$y=\sqrt{x-(+6)}$	$y=\sqrt{x-5}$	$y=\sqrt{x-h}$ $h>0$
Vertical Stretch	$y=3\sqrt{x}$	$y=5\sqrt{x}$	$y=a\sqrt{x}$ $a>1$
Vertical Compression	$y=\frac{2}{3}\sqrt{x}$	$y=\frac{1}{5}\sqrt{x}$	$y=a\sqrt{x}$ $0<a<1$
Reflection	$y=-\sqrt{x}$	$y=-5\sqrt{x}$	$y=a\sqrt{x}$ $a<0$

2. **Describe the role of *a*.**
   $|a|$ is the vertical stretch or compression factor. If a<0, the graph is reflected across the x-axis.

3. **Describe the role of *h*.**
   h is the horizontal translation.

4. **Describe the role of *k*.**
   k is the vertical translation.

5. **Using *x, a, h* , and *k*, write an equation that could be used to summarize the transformations to the square root function.**
   $y = a\sqrt{x-h} + k$ *or, using function notation*, $f(x) = a\sqrt{x-h} + k$

6. **Revisiting the bottle bounce investigation, describe the transformation to the square root parent function that represents your data.**
   Answers may vary.

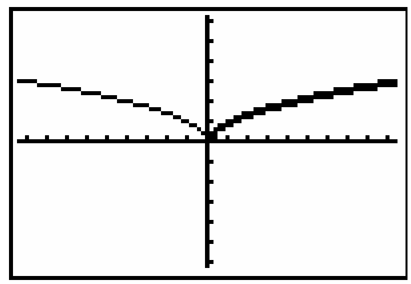
<u>Part 5 (Optional Extension)</u>: Investigating the Coefficient of *x*

Leader notes: Horizontal stretches, compressions, and reflections are explored in Precalculus. However, some participants may respond that the transformation form is $f(x) = a\sqrt{\frac{1}{b}(x-h)} + k$ with $|b|$ being the horizontal stretch or compression factor, and that if $b < 0$, the graph is reflected across the y-axis.

1. Using $f(x) = a\sqrt{\frac{1}{b}(x-h)} + k$, predict the changes in the parent function for the following functions. Then check with your graphing calculator.

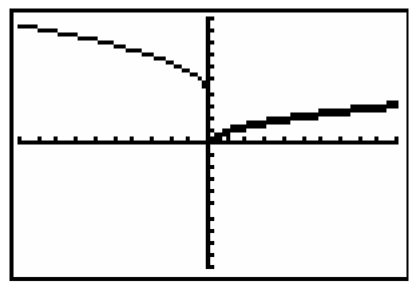
a. $f(g) = \sqrt{-x}$

The graph of the parent function will be reflected across the y-axis.



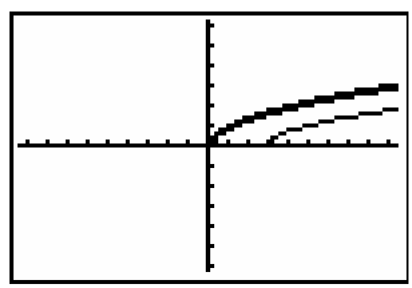
b. $f(g) = \sqrt{-3x} + 4$

The graph of the parent function will be reflected across the y-axis, horizontally compressed by a factor of $\frac{1}{3}$, and then translated 4 units up.



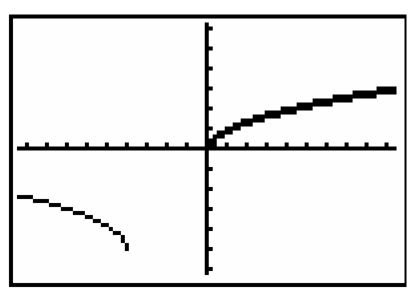
c. $f(g) = \sqrt{\frac{1}{2}(x-3)}$

The graph of the parent function will be horizontally stretched by a factor of 2 and translated 3 units to the right.



d. $f(g) = 2\sqrt{-\frac{1}{3}(x+4)} - 5$

The graph of the parent function will be vertically stretched by a factor of 2, reflected across the y-axis, horizontally stretched by a factor of 3, translated 4 units left, and translated 5 units down. (Whew!)



2. What can you summarize about transformations of the square root parent function as a result of changes to $\frac{1}{b}$?

$|b|$ is the horizontal stretch or compression factor. If $b<0$, the graph is reflected across the y-axis.

<u>Part 6 (Optional Extension)</u>: Connecting the Roles of *a, h,* and *k* in Square Root and Quadratic Functions

Display Transparency A. Ask participants to find the inverse of each equation algebraically.

Equation 1 $y = 3\sqrt{x-5}+6$

$$x = 3\sqrt{y-5}+6$$
$$x-6 = 3\sqrt{y-5}+6-6$$
$$x-6 = 3\sqrt{y-5}$$
$$\frac{x-6}{3} = \frac{3\sqrt{y-5}}{3}$$
$$\frac{x-6}{3} = \sqrt{y-5}$$
$$\left(\frac{x-6}{3}\right)^2 = \left(\sqrt{y-5}\right)^2$$
$$\left(\frac{x-6}{3}\right)^2 = y-5$$
$$\left(\frac{x-6}{3}\right)^2 + 5 = y-5+5$$
$$\left(\frac{x-6}{3}\right)^2 + 5 = y$$

or

$$y = \left(\frac{1}{3}\right)^2 (x-6)^2 + 5$$

Equation 2 $y = 7(x-2)^2 + 4$

$$x = 7(y-2)^2 + 4$$
$$x - 4 = 7(y-2)^2 + 4 - 4$$
$$x - 4 = 7(y-2)^2$$
$$\frac{x-4}{7} = \frac{(y-2)^2}{7}$$
$$\frac{x-4}{7} = (y-2)^2$$
$$\pm\sqrt{\frac{x-4}{7}} = \sqrt{(y-2)^2}$$
$$\pm\sqrt{\frac{x-4}{7}} = y - 2$$
$$\pm\sqrt{\frac{x-4}{7}} + 2 = y - 2 + 2$$
$$\pm\sqrt{\frac{x-4}{7}} + 2 = y$$

or

$$y = \frac{1}{\sqrt{7}}\left(\sqrt{x-4}\right) + 2 \quad \text{or} \quad y = -\frac{1}{\sqrt{7}}\left(\sqrt{x-4}\right) + 2$$

1. **Find the inverse of Equation 1.**

2. **Numerically and graphically compare and contrast Equation 1 and its inverse.**
   Answers will vary but should demonstrate an understanding that the inverse of the square root equation in Example 1 is a quadratic function that is translated 6 units right; the square root function is shifted 6 units up. The quadratic function is shifted 5 units up; the square root function is shifted 5 units to the right. The quadratic equation is vertically compressed by a factor of $\left(\frac{1}{3}\right)^2$ *while the square root equation is vertically stretched by a factor of 3.*

3. **Find the inverse of Equation 2.**

4. **Numerically and graphically compare and contrast Equation 2 and its inverse.**
   Answers will vary but should demonstrate an understanding that the inverse of Equation 2 is a square root equation in which the vertical and horizontal translations are the reverse of the values in the original quadratic equation. Equation 2 is vertically stretched by a factor of 7 while its inverse, the square root equation, is compressed vertically by a factor of $\frac{1}{\sqrt{7}}$.

5. Find the inverse of $y = a\sqrt{(x-h)} + k$.

Possible response is shown on the following page.

6. Find the inverse of $y = a(x-h)^2 + k$.

Possible response is shown on the following page.

Display Transparency B. Ask participants to find the inverse of each equation algebraically.

$$y = a\sqrt{x-h} + k$$

$$x = a\sqrt{y-h} + k$$

$$x - k = a\sqrt{y-h} + k - k$$

$$\frac{x-k}{a} = \frac{a\sqrt{y-h}}{a}$$

$$\left(\frac{x-k}{a}\right)^2 = \left(\sqrt{y-h}\right)^2$$

$$\left(\frac{x+k}{a}\right)^2 = y - h$$

$$\left(\frac{x+k}{a}\right)^2 + h = y - h + h$$

$$y = \left(\frac{x+k}{a}\right)^2 + h \text{ or } y = \left(\frac{1}{a}\right)^2 (x+k)^2 + h$$

$$y = a(x-h)^2 + k$$

$$x = a(y-h)^2 + k$$

$$x - k = a(y-h)^2 + k - k$$

$$\frac{x-k}{a} = \frac{a(y-h)^2}{a}$$

$$\frac{x-k}{a} = (y-h)^2$$

$$\pm\sqrt{\frac{x-k}{a}} = \pm\sqrt{(y-h)^2}$$

$$\pm\sqrt{\frac{x-k}{a}} = y - h$$

$$\pm\sqrt{\frac{x-k}{a}} + h = y - h + h$$

$$\pm\sqrt{\frac{x-k}{a}} + h = y$$

$$y = \frac{\sqrt{x-k}}{\sqrt{a}} + h \text{ or } y = \frac{1}{\sqrt{a}}\left(\sqrt{x-k}\right) + h \quad \text{OR} \quad y = -\frac{\sqrt{x-k}}{\sqrt{a}} + h \text{ or } y = -\frac{1}{\sqrt{a}}\left(\sqrt{x-k}\right) + h$$

7. Summarize the relationship between *h* and *k* in the square root transformation form and *h* and *k* in the quadratic transformation (vertex) form.
The h and k are reversed. That is, the h in the square root function becomes the k in the quadratic function and vice versa. The k in the square root function becomes the h in the quadratic function and vice versa.

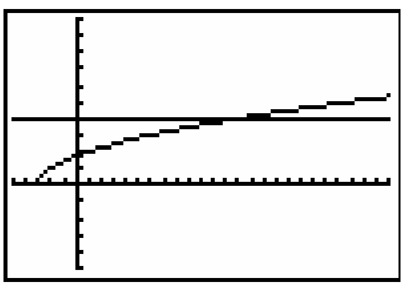
8. Summarize the relationship between *a* in the square root transformation form and *a* in the quadratic transformation (vertex) form.
The "a" in the square root function is inversely proportional to the square of "a" in the quadratic function. The "a" in the quadratic function is inversely proportional to the square root of "a" in the square root function.

Part 7: Solving Square Root Equations and Inequalities

Leader Note:
In this part of the professional development participants investigate solutions to square root equations and inequalities using graphs, tables, and algebraic representations.

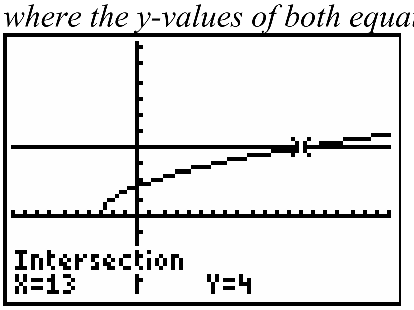
1. Consider the system of equations $y = \sqrt{x+3}$ **and** $y = 4$**.**

a) Graph the system and sketch the graph. What are the domain and range for each function in this system?



For the function $y = \sqrt{x+3}$*, the domain is all real numbers greater than –3, and the range is all y-values greater than 0. For the function* $y = 4$*, the domain is all real numbers, and the range is 4.*

b) How can you determine the solution to this system of equations graphically or tabularly?
I can find the point of intersection graphically, and/or I can find the values in the table where the y-values of both equations are the same for an x-value.



X	Y1	Y2
8	3.3166	4
9	3.4641	4
10	3.6056	4
11	3.7417	4
12	3.873	4
13	4	4
14	4.1231	4

X=13

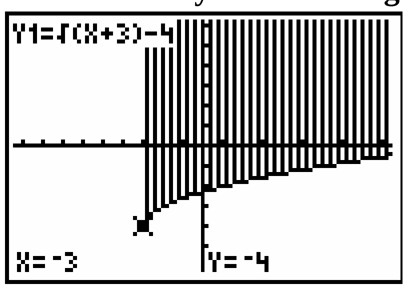
c) What are the coordinates of the point that is a solution for this system?
The solution is (13, 4).

d) How can you use the transitive property to write this system as one equation?
$\sqrt{x+3}=4$

e) How can you solve this equation algebraically?

$$\begin{aligned}&\sqrt{x+3}=4\\&\left(\sqrt{x+3}\right)^2=4^2\\&x+3=16\\&x+3-3=16-3\\&x=13\end{aligned}$$

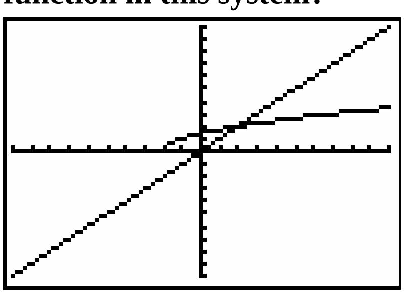
f) What would be your solution set if you had been given $\sqrt{x+3}\geq 4$ **?**



The solution is all real numbers where x is greater than –3 and y is greater than –4.

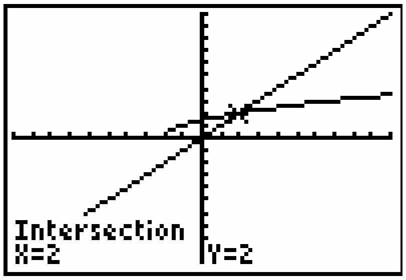
2. Consider the system of equations $y=\sqrt{x+2}$ **and** $y=x$ **.**

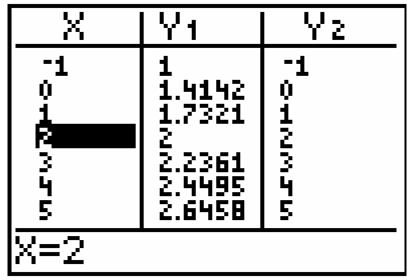
a) Graph the system and sketch the graph. What are the domain and range for each function in this system?



For the function $y=\sqrt{x+2}$ ***, the domain is all real numbers greater than –2, and the range is all y-values greater than 0. For the function*** $y=x$ ***, the domain is all real numbers, and the range is all real numbers.***

b) How can you determine the solution to this system graphically or tabularly?
I can find the point of intersection graphically, and/or I can find the values in the table where the y-values of both equations are the same for an x-value.





X	Y_1	Y_2
-1	1	-1
0	1.4142	0
1	1.7321	1
2	2	2
3	2.2361	3
4	2.4495	4
5	2.6458	5

X=2

c) What are the coordinates of the point that is a solution for this system of equations?
The solution is (2, 2).

d) How can you use the transitive property to write this system as one equation?
$\sqrt{x+2} = x$

e) How can you solve this equation algebraically?

$$\sqrt{x+2} = x$$

$$\left(\sqrt{x+2}\right)^2 = x^2$$

$$x+2 = x^2$$

$$0 = x^2 - x - 2$$

$$0 = (x-2)(x+1)$$

$$x = 2 \quad x = -1$$

f) Are both solutions valid? Why or why not?
When I substitute the solutions into the equation, I can verify that 2 is a valid solution. However, –1 produced an extraneous root. It does not give me valid solution.

$$\sqrt{x+2} = x \qquad \sqrt{x+2} = x$$

$$\sqrt{2+2} = 2 \qquad \sqrt{-1+2} = -1$$

$$\sqrt{4} = 2 \qquad \sqrt{1} = -1$$

$$2 = 2 \qquad 1 \neq -1$$

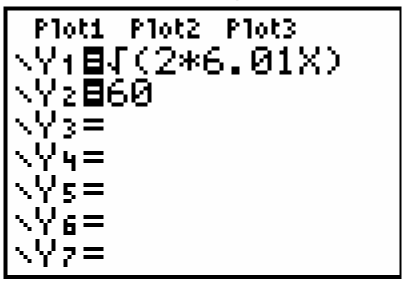
This is also demonstrated graphically and tabularly in ***b)*** *above.*

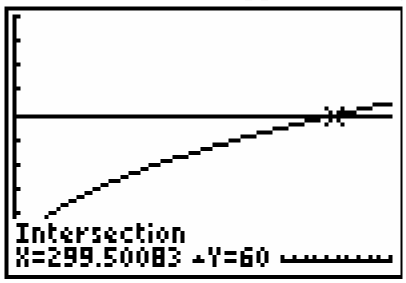
Leader notes:
The purpose of Question 3 is to allow participants to investigate a square root function in a real world context. At the end of the investigation they should be more willing to consider solving square root equations with tables and graphs in addition to the traditional symbolic solution.

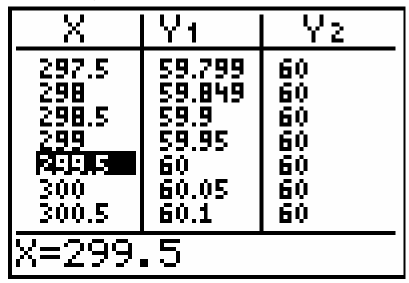
3. Choose one of the following problems. Work with your group to find the solutions(s). Justify your answer. Use chart paper to display your work.

A. Jim is an accident investigator who was asked to determine whether a driver's excessive speed was a factor in a traffic accident. The traditional equation used to determine the speed at which a vehicle was traveling at the onset of the skid is $V_s = \sqrt{2aS_s}$, where *a* is the deceleration force of gravity times friction, and S_s is the length of the skid marks. If the speed limit is 60 mph and the skid marks are 225 ft. long, was the driver exceeding the speed limit? (Use 6.01 for *a*.) What is the maximum length skid mark that would have exonerated the driver? Justify your answer.

No. The driver was traveling at approximately 52 mph.
The maximum length skid mark would have been approximately 299.5 ft.







$$60 = \sqrt{2 * 6.01x}$$

$$60^2 = \left(\sqrt{2 * 6.01x}\right)^2$$

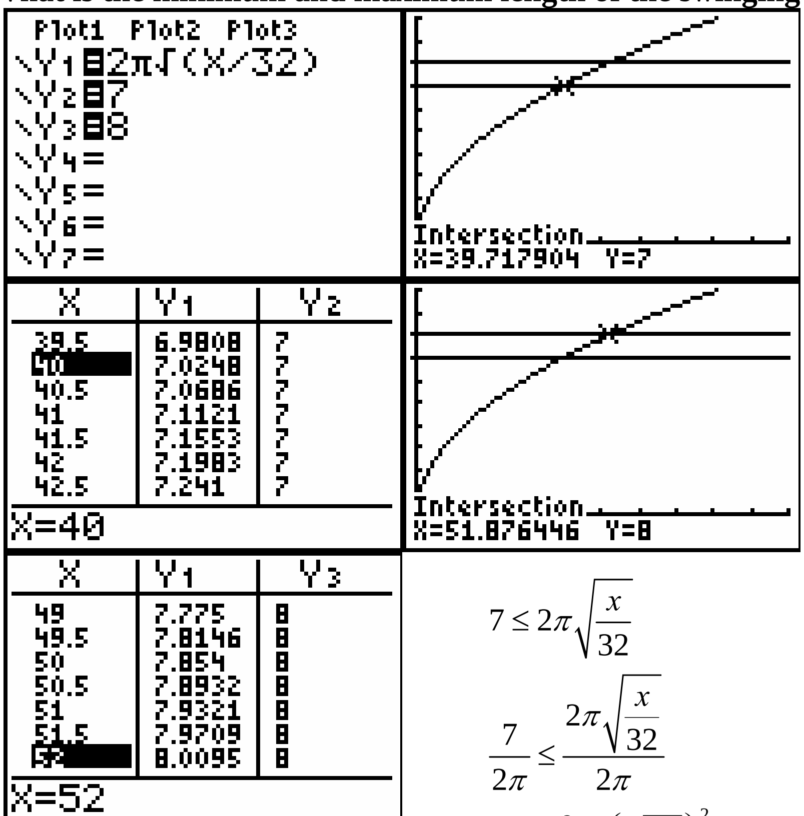
$$3600 = 12.02x$$

$$299.5 \approx x$$

B. At Thalia's favorite amusement park, there is a ride called the "Pirate Ship." People sit in what looks like a huge ship. The "ship" then swings back and forth. Thalia notices that it takes somewhere between 7 and 8 seconds for the ride to make one complete swing back and forth.

The function that represents the time in seconds of one complete swing, *t*, based on the height of the swinging bar, *h*, in feet, is $t = 2\pi\sqrt{\frac{h}{32}}$ **.**

What is the minimum and maximum length of the swinging bar?



$$7 \le 2\pi\sqrt{\frac{x}{32}}$$

$$\frac{7}{2\pi} \le \frac{2\pi\sqrt{\frac{x}{32}}}{2\pi}$$

$$\left(\frac{7}{2\pi}\right)^2 \le \left(\sqrt{\frac{x}{32}}\right)^2$$

$$(32)\left(\frac{7}{2\pi}\right)^2 \le \frac{x}{32}(32)$$

$$39.71790399 \le x$$

$$8 \ge 2\pi\sqrt{\frac{x}{32}}$$

$$\frac{8}{2\pi} \ge \frac{2\pi\sqrt{\frac{x}{32}}}{2\pi}$$

$$\left(\frac{8}{2\pi}\right)^2 \ge \left(\sqrt{\frac{x}{32}}\right)^2$$

$$(32)\left(\frac{8}{2\pi}\right)^2 \ge \frac{x}{32}(32)$$

$$51.87644602 \ge x$$

The swinging bar is between 39.71 and 51.88 ft. long.

C. Arnie was taking a picture from the window of his apartment. Unfortunately, he dropped the camera, which landed on the ground at least 2 seconds later. The equation that models the time, *t*, it takes for an object to fall *h* meters is $t=\sqrt{\dfrac{2h}{9.81}}$ **.**

From what height did Arnie drop the camera?

$$2=\sqrt{\frac{2h}{9.81}}$$

$$2^2=\left(\sqrt{\frac{2h}{9.81}}\right)^2$$

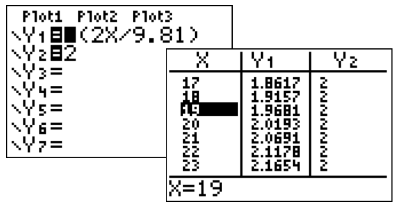
$$4=\frac{2h}{9.81}$$

$$9.81\bullet 4=\frac{2h}{9.81}\bullet 9.81$$

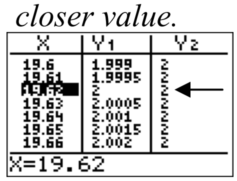
$$39.24=2h$$

$$19.62=h$$

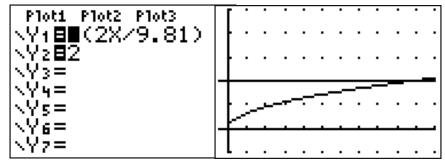
He dropped the camera from a height of 19.62 meters.



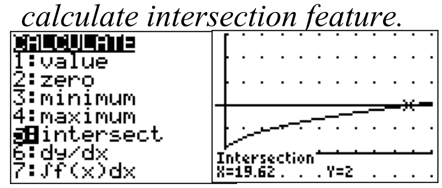
The height is between 19 and 20, so adjust ΔTbl to get a closer value.



He dropped the camera from a height of about 19.62 meters.

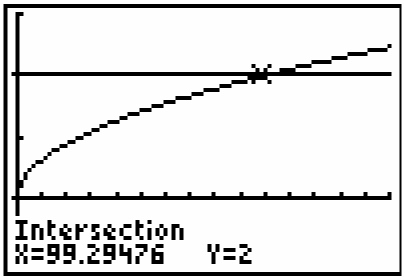


You can trace to the point of intersection, or use the calculate intersection feature.



For a time of 2 seconds, the height is 19.62 meters.

D. Sharon's mother bought a grandfather clock and asked Sharon to determine how long the pendulum must be so the clock keeps accurate time. Sharon found the formula $t = 2\pi\sqrt{\frac{L}{g}}$, where *t* is the time for one complete swing of the clock pendulum, *L* is the length of the pendulum, and *g* is acceleration due to gravity (which is 980 cm/sec^2). Since the time for a complete swing of the pendulum of a grandfather clock must be 2 seconds, how long should the pendulum be?



X	Y1	Y2
97	1.9768	2
97.5	1.9818	2
98	1.9869	2
98.5	1.992	2
99	1.997	2
99.5	2.0021	2
100	2.0071	2

X=99.5

$$t = 2\pi\sqrt{\frac{L}{g}}$$

$$2 = 2\pi\sqrt{\frac{L}{980}}$$

$$\frac{2}{2\pi} = \frac{2\pi\sqrt{\frac{L}{980}}}{2\pi}$$

$$\frac{1}{\pi} = \sqrt{\frac{L}{980}}$$

$$\left(\frac{1}{\pi}\right)^2 = \left(\sqrt{\frac{L}{980}}\right)^2$$

$$\left(\frac{1}{\pi}\right)^2 = \frac{L}{980}$$

$$(980)\left(\frac{1}{\pi}\right)^2 = \frac{L}{980}(980)$$

$$99.29 \approx L$$

The length of the pendulum should be approximately 99.29 cm.

Explain

Leaders' Note: The Maximizing Algebra II Performance (MAP) professional development is intended to be an extension of the ideas introduced in Mathematics TEKS Connections (MTC). Throughout the professional development experience, we allude to components of MTC such as the Processing Framework Model, the emphasis of making connections among representations, and the links between conceptual understanding and procedural fluency.

Debriefing the Experience:

1. **What concepts did we explore in the previous set of activities? How were they connected?**
   Responses may vary. Participants should observe that investigating the physical and graphical representations of square root functions enhances understanding of the concept of square root. Square root equations and inequalities can be solved by graphing the equations and inequalities as systems.

   Facilitation Questions
   - **How would you describe the conceptual progression in this explore phase?**
     Answers may vary. Possible response: We began by exploring a physical representation of the square root function and then progressed to graphical, tabular, and symbolic representations.
   - **What role did representations play in the conceptual progression?**
     Answers may vary. Possible response: We saw the connections among the physical model and the corresponding graphs, tables, and symbolic representations.

2. **What procedures did we use to describe square root functions? How are they related?**
   Tabular, graphical, and symbolic procedures were all used throughout the Explore phase. Ultimately, they are all connected through the numerical relationships used to generate them.

   Facilitation Questions
   - **How would you describe the procedural progression in this explore phase?**
     Answers may vary. Possible response: After collecting the data, we tried several function rules and found one that approximated our data. Then we connected it to both the graph and table. We were able to expand those procedures to form generalizations about transformations and solutions to square root functions.
   - **What role did representations play in the procedural progression?**
     Answers may vary. Possible response: We saw that we could solve square root functions using graphs, tables, and symbolic representations.

3. **What knowledge from Algebra I do students bring about square root functions?**
   Square root is not a student expectation until Algebra II, so students may not have had prior instruction on the square root of a number or square root functions.

4. **After working with square root functions in Algebra II, what are students' next steps in Precalculus or other higher mathematics courses?**
   According to the Precalculus TEKS, students will expand their understanding of square root functions. They will be expected to apply basic transformations and compositions with square root functions, including $\left|f(x)\right|$, *and* $f(|x|)$, *to the parent functions.*

Anchoring the Experience:

5. **Distribute to each table group a poster-size copy of the Processing Framework Model.**

6. **Ask each group to respond to the question:**

   Where in the processing framework would you locate the different activities from the Explore phase?

7. **Participants can use one color of sticky notes to record their responses. In future Explain phases, participants will use other colors of sticky notes to record their responses.**

Horizontal Connections within the TEKS

8. **Direct the participants' attention to the second layer in the Processing Framework Model: Horizontal Connections among Strands.**

9. **Prompt the participants to study the Algebra II TEKS and record those TEKS that connect to this Explore/Explain cycle. Prompt participants to attend to both the knowledge statements as well as the student expectations.**

10. **Invite each table group to share two connections that they found and record them so that they are visible to the entire group.**

Vertical Connections within the TEKS

11. **Direct the participants' attention to the third layer in the Processing Framework Model: Vertical Connections across Grade Levels.**

12. **Prompt the participants to study the Algebra I, Geometry, Math Models, and Precalculus TEKS and record those TEKS that connect to this Explore/Explain cycle. Prompt participants to attend to both the knowledge statements as well as the student expectations.**

13. **Invite each table group to share two connections that they found, recording so that the entire large group may see.**

14. **Provide each group of participants with a clean sheet of chart paper. Ask them to create a "mind map" for the mathematical term of "square root function."**
    See next page for possible response.

15. Provide an opportunity for each group to share their mind maps with the larger group. Discuss similarities, differences, and key points brought forth by participants.

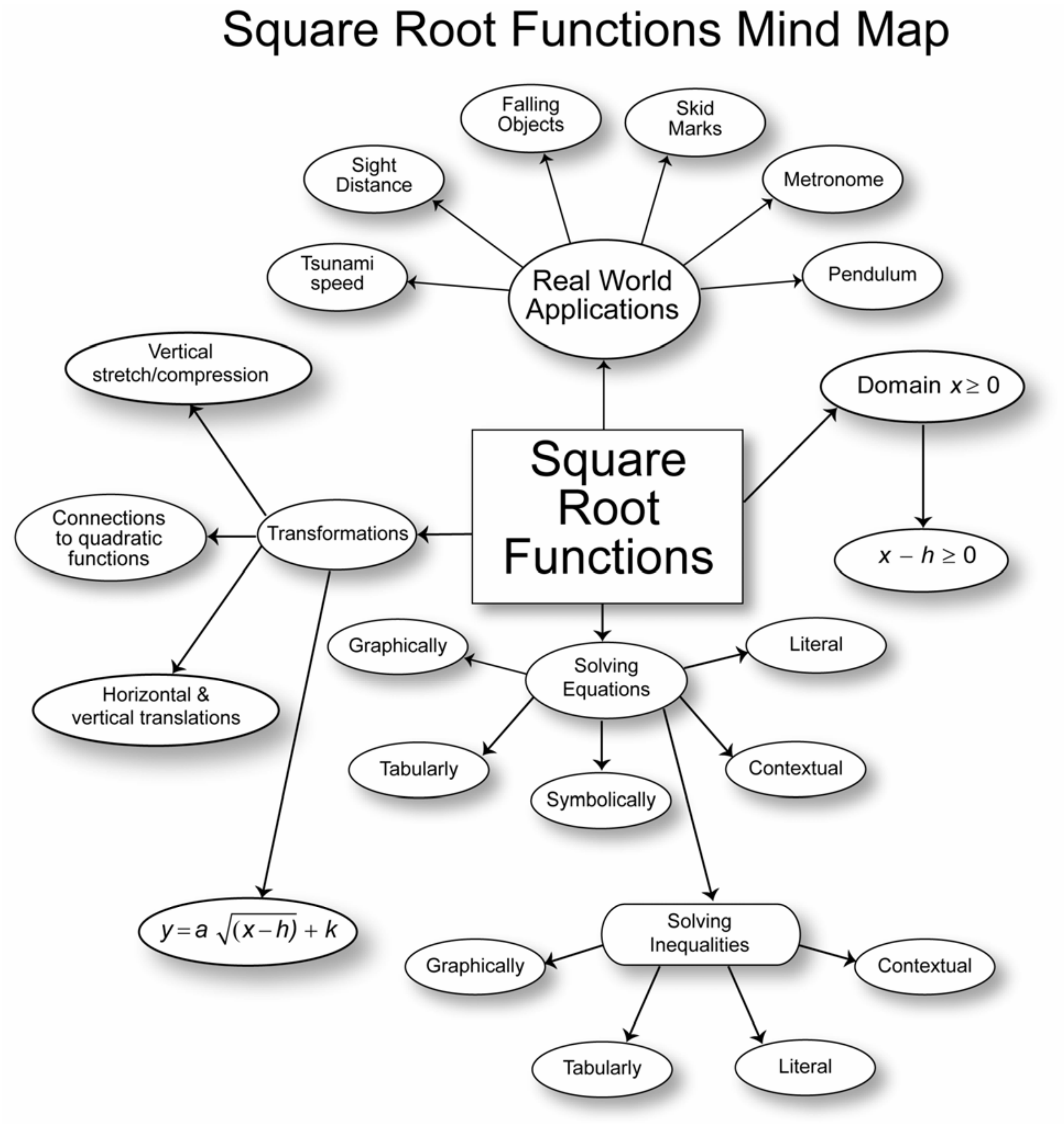
16. Distribute the vocabulary organizer template to each participant. Ask participants to construct a vocabulary model for the term "square root functions."

17. When participants have completed their vocabulary models, ask participants to identify strategies from their experiences so far in the professional development that could be used to support students who typically struggle with Algebra II topics.

Note to Leader: *You may wish to have each small group brainstorm a few ideas first, then share their ideas with the large group while you record their responses on a transparency or chart paper.*

18. How would this lesson maximize student performance in Algebra II for teaching and learning the mathematical concepts and procedures associated with square root functions?

Responses may vary. Anchoring procedures within a conceptual framework helps students understand what they are doing so that they become more fluent with the procedures required to accomplish their tasks. Problems present themselves in a variety of representations; providing students with multiple procedures to solve a given problem empowers students to solve the problem more easily.


Square Root Functions Mind Map
Falling Objects
Skid Marks
Sight Distance
Metronome
Tsunami speed
Real World Applications
Pendulum
Vertical stretch/compression
Domain $x \geq 0$
Square Root Functions
Connections to quadratic functions
Transformations
$x - h \geq 0$
Graphically
Solving Equations
Literal
Horizontal & vertical translations
Tabularly
Symbolically
Contextual
$y = a\sqrt{(x-h)} + k$
Solving Inequalities
Graphically
Contextual
Tabularly
Literal

Elaborate

Leaders' Note: *In this phase, participants will extend their learning experiences to their classroom.*

1. **Distribute the 5E Student Lesson planning template. Ask participants to think back to their experiences in the Explore phase. Pose the following task:**

   What might a student-ready 5E lesson on square root functions look like?
   - **What would the Engage look like?**
   - **Which experiences/activities would students explore firsthand?**
   - **How would students formalize and generalize their learning?**
   - **What would the Elaborate look like?**
   - **How would we evaluate student understanding of inverses of relations/functions?**

2. **After participants have recorded their thoughts, direct them to the student lesson for square root functions. Allow time for participants to review lessons.**

3. **How does this 5E lesson compare to your vision of a student-centered 5E lesson?**
   Responses may vary.

4. **How does this lesson help remove obstacles that typically keep students from being successful in Algebra II?**
   By connecting the concept of square root to a physical model, students gain a better understanding of the square root function and the limitations of using a linear function to describe the model. By solving square root equations as systems and through graphing, solutions can be verified and connected to the algebraic processes commonly used to find those solutions. Students can use alternate methods with which to solve meaningful problems.

5. **How does this lesson maximize your instructional time and effort in teaching Algebra II?**
   Taking time to create a solid conceptual foundation reduces the need for re-teaching time and effort and increases student participation in the learning process. Conceptual connections to algebraic process strengthen the understanding of square root functions and mirror the links among multiple representations.

6. **How does this lesson maximize student learning in Algebra II?**
   Using multiple representations and foundations for functions concepts allows students to make connections among different ideas. These connections allow students to apply their learning to new situations more quickly and readily.

7. **How does this lesson accelerate student learning and increase the efficiency of learning?**
   Foundations for functions concepts such as function transformations transcend all kinds of functions. A basic toolkit for students to use when working with functions allows students to rethink what they know about linear and quadratic functions while they are learning concepts and procedures associated with other function families.

8. **Read through the suggested strategies on Strategies that Support English Language Learners. Consider the possible strategies designed to increase the achievement of English language learners.**
   As participants read through the strategies that support English language learners and strategies that support students with special needs, they may notice that eight of the ten strategies are the same. The intention is to underscore effective teaching practices for all students. However, English language learners have needs specific to language that students with special needs may or may not have. The two strategies that are unique to the English language learners reflect an emphasis on language. Students with special needs may have prescribed modifications and accommodations that address materials and feedback. Students with special needs often benefit from progress monitoring with direct feedback and adaptation of materials for structure and/or pacing. A system of quick response is an intentional plan to gather data about a student's progress to determine whether or not the modification and (or) accommodation are (is) having the desired effect. The intention of the strategies is to provide access to rigorous mathematics and support students as they learn rigorous mathematics.

9. **What evidence of these strategies do you find in this portion of the professional development?**
   Responses may vary. ***Note:*** *Some strategies reflect teacher behaviors. The presenter may need to prompt participants to consider how the professional development materials support the needed teacher behaviors. For example, a student lesson may outline a structured approach for exploration so that the activity is non-threatening. This contributes to the teacher's ability to create an emotionally safe environment for learning. Tools such as the CBR and graphing calculator can be used to communicate about and solve problem situations involving square root functions.*

10. **Which strategies require adaptation of the materials in this portion of the professional development?**
    Responses may vary. Most of the strategies are incorporated throughout the materials.

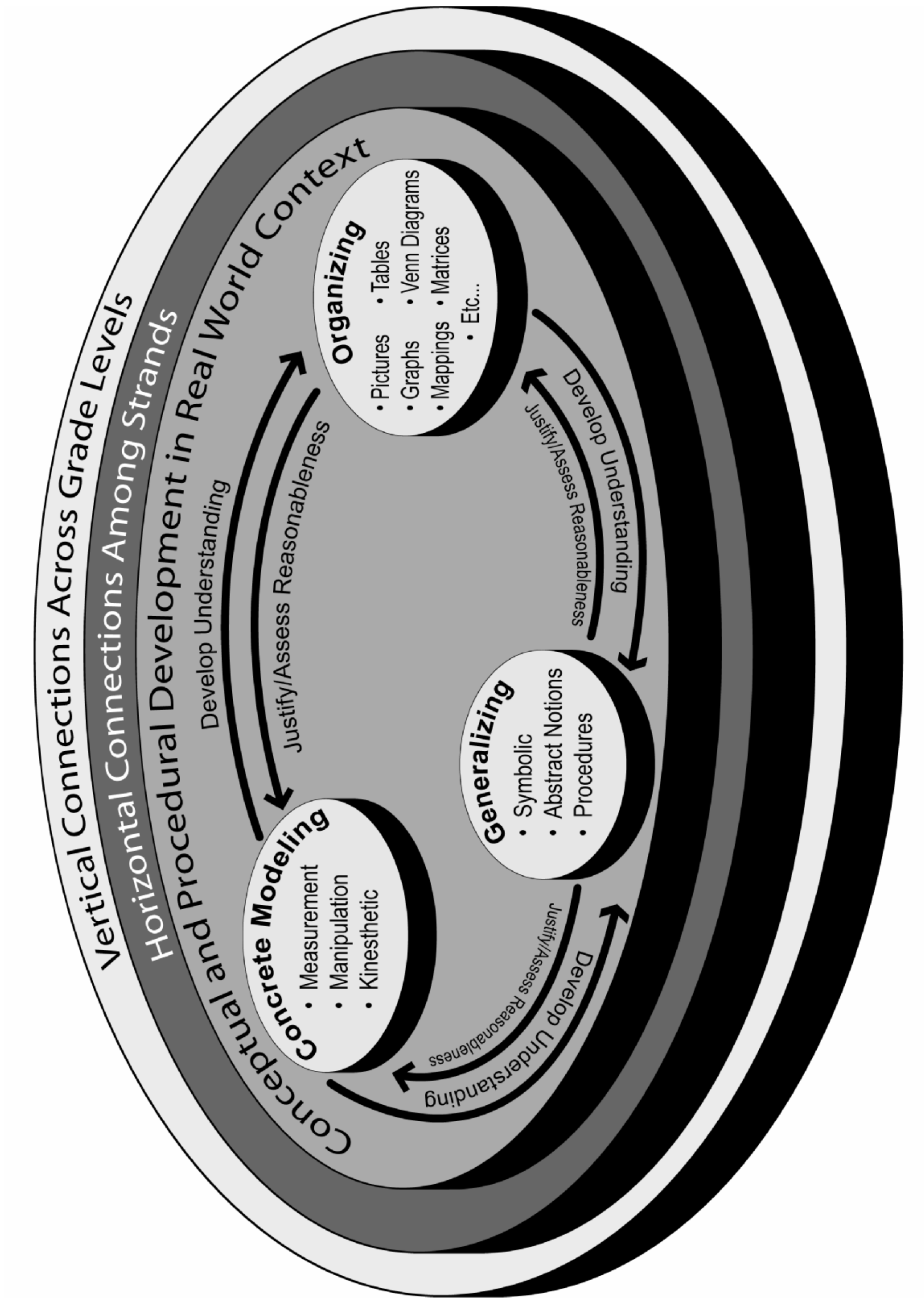
11. **Read through the suggested strategies on Strategies that Support Students with Special Needs. Consider the possible strategies designed to increase the achievement of students with special needs.**

12. **What evidence of these strategies do you find in this portion of the professional development?**
    Responses may vary. ***Note:*** *Some strategies reflect teacher behaviors. The presenter may need to prompt participants to consider how the professional development materials support the needed teacher behaviors. For example, a student lesson may outline a structured approach for exploration so that the activity is non-threatening. This contributes to the teacher's ability to create an emotionally safe environment for learning. Tools such as the CBR and graphing calculator can be used to communicate about and solve problem situations involving square root functions.*

13. Which strategies require adaptation of the materials in this portion of the professional development?

Responses may vary. Most of the strategies are incorporated throughout the materials. Some materials may need to be modified for format or structure.

Processing Framework Model



Vocabulary Organizer

My definition

Personal Association

Example

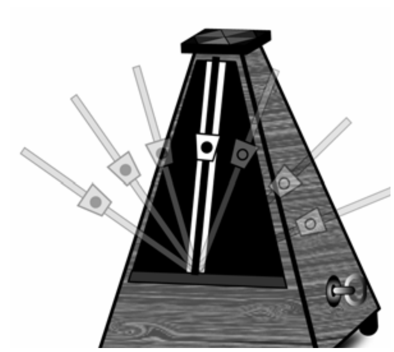
Non-Example

Sample vocabulary organizer for square root functions

My definition

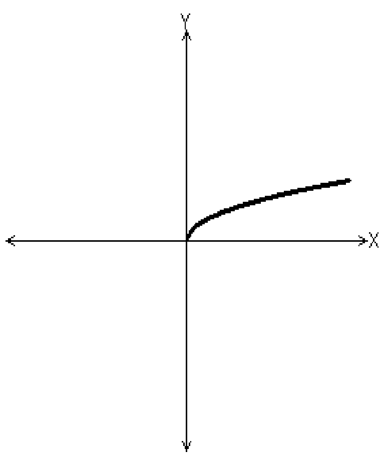
The inverse of the positive side of the quadratic parent function

Personal Association

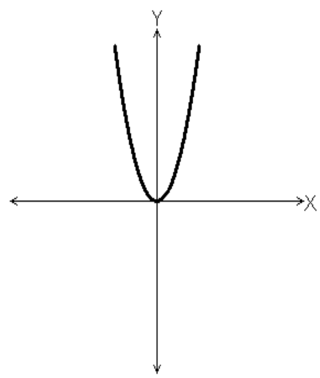


Square root function

Example



Non-Example



5E Student Lesson Planning Template

Description	Activity
Engage The activity should be designed to generate student interest in a problem situation and to make connections to prior knowledge. The instructor initiates this stage by asking meaningful questions, posing a problem to be solved, or by showing something intriguing.	
Explore The activity should provide students with an opportunity to become actively involved with the key concepts of the lesson through a guided exploration requiring them to probe, inquire, and question. The instructor actively monitors students as they interact with each other and the activity.	
Explain Students collaboratively begin to sequence events/facts from the investigation and communicate these findings to each other and the instructor. The instructor, acting in a facilitation role, formalizes student findings by providing further explanations and additional meaning or information, such as correct terminology.	
Elaborate Students extend, expand, or apply what they have learned in the first three stages and connect this knowledge with prior learning to deepen understanding. Instructors can use the Elaborate stage to verify students' understandings.	
Evaluate Evaluation occurs throughout students' learning experiences. More formal evaluation can be conducted at this stage. Instructors can determine whether the learner has reached the desired level of understanding the key ideas and concepts.	

Strategies that Support English Language Learners (ELL)

Strategy	Explore, Explain, Elaborate 2
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Be conscious of tone and diction. Speak slowly and distinctly.	
Incorporate language skills (reading, writing, speaking, and listening) into instruction.	

Strategies that Support Students with Special Needs

Strategy	Explore, Explain, Elaborate 2
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Use a system of quick response to needs and accommodations including progress monitoring to inform instruction.	
Accommodate materials for format, structure, sequence, etc. as needed.	

Transparency A

Equation 1

$$y = 3\sqrt{x-5} + 6$$

Equation 2

$$y = 7(x-2)^2 + 4$$

Transparency B

$$y = a\sqrt{x-h} + k$$

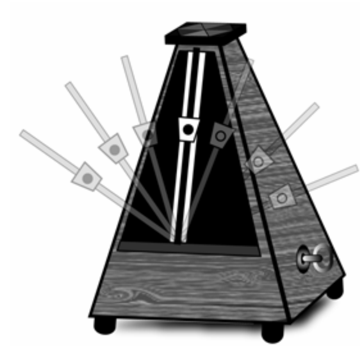
$$y = a(x-h)^2 + k$$

Participant Pages: Square Root Functions

Explore

<u>Part 1</u>: Setting the Stage

A metronome frequently is used in music to mark exact time using a repeated tick. The frequency of the ticks varies in musical terms from slow *(largo,* about 40-60 beats per minute) to fast *(presto,* 168-208 beats per minute). Individual instrumentalists, choirs, bands, and orchestras all use metronomes to ensure that the beat of the music is consistent with the instructions of the composer and does not unintentionally speed up or slow down while the piece is being played.



What do you notice about the frequency of the ticks and setting of the weight as the music is being played?

<u>Part 2</u>: Modeling and Gathering Data

Divide participants into groups of 4. Each person in the group has a job.

Materials manager: Gets the necessary materials, directs the team in setting up the investigation, holds the pencil with the suspended bottle, and shortens the spring when needed.

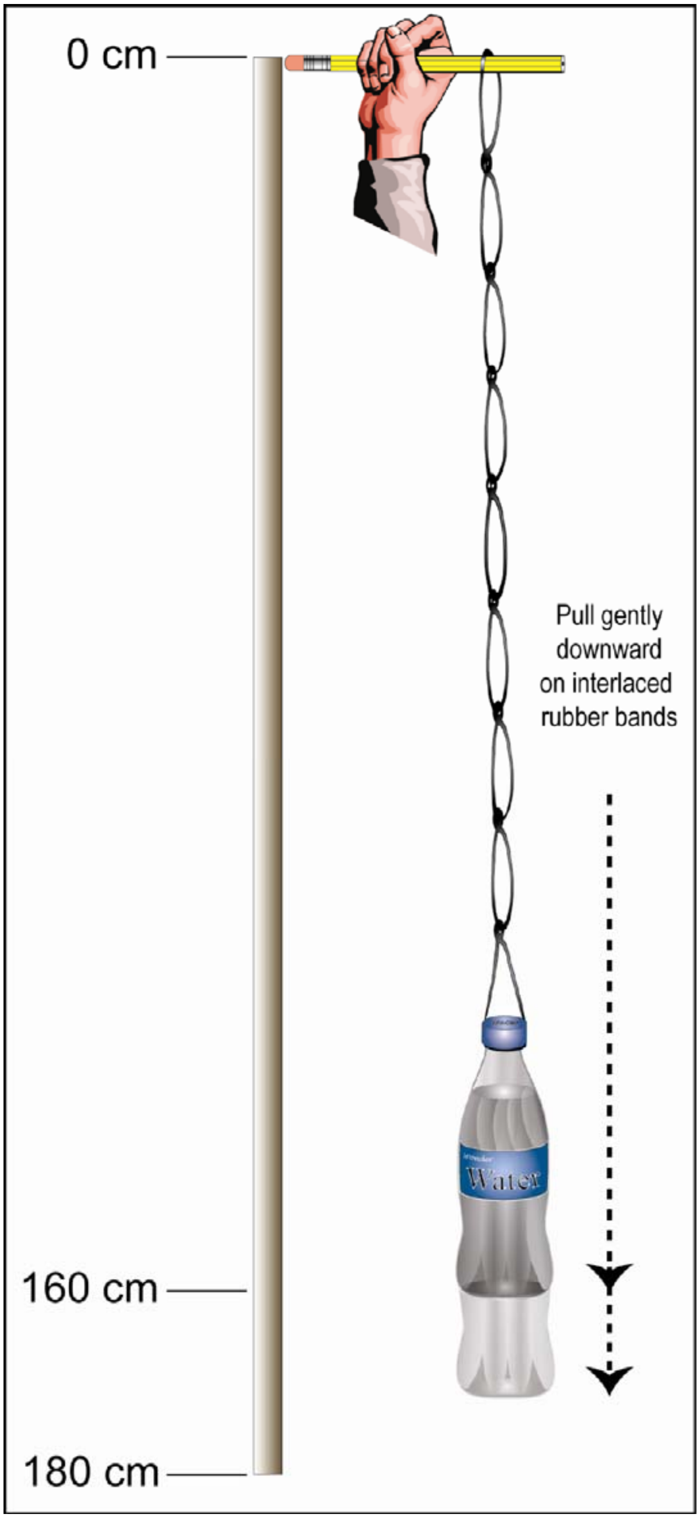
Measures manager: Measures the distance from the pencil to the bottom of the bottle for each length, initiates the bounce by pulling the suspended bottle down an additional 10 centimeters and counts the bounces (10 at each height).

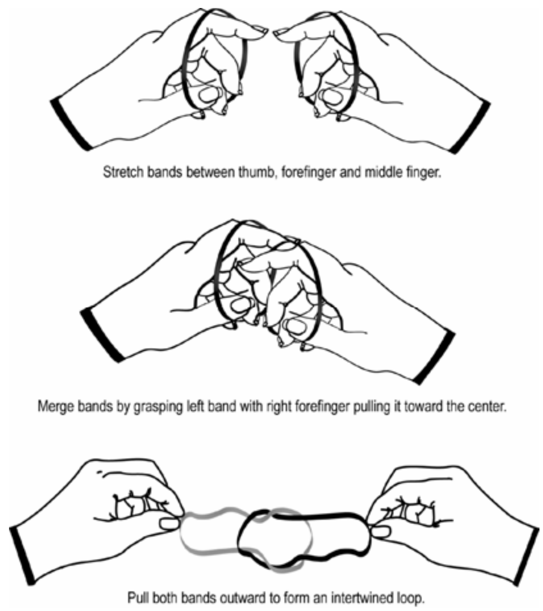
Time manager: Uses a stop watch to determine the length of each 10-bounce period of time. The time starts when the bottle is released by the measures manager and ends when the bottle completes its 10^{th} bounce.

Data manager: Records the necessary measurements in the table and shares the data with the team.

Set-up Instructions

Step 1. The materials manager should get the necessary materials and ask two of the team members to secure the tape measure or meter sticks against the wall. The tape measure or meter sticks should be positioned perpendicular to the floor so that the "zero end" is at 180-200 centimeters above the floor.





Step 2. While the tape measure or meter sticks are being positioned, the materials manager and remaining team member(s) build the rubber band spring by looping rubber bands together until the length of the spring is about 1 meter. This task will go more quickly if each person makes about half of the spring. Then the pieces can be joined.

Step 3. Secure one end of the rubber band spring to the pencil and the other around the neck of the bottle. It also works to remove the cap, insert the end of the spring in the bottle, and screw the cap back on.

Step 4. The measures manager secures the spring so that it is approximately 160 centimeters in length. After the bottle remains motionless for a few seconds, he should measure the actual length of the spring. The length of the spring includes the length of the rubber band and the length of the bottle.

Step 5. The measures manager pulls the bottle downward about 10 cm and releases it.

Step 6. The time manager starts the stopwatch when it is released and stops it at the end of 10 complete bounces. It may be helpful to have all team members count aloud together.

Step 7. The data manager records the number of seconds in the table under Trial 1 for 160 cm. *Hint: It is more meaningful to start with the spring fully extended and to shorten the spring than to begin at the top and work down.*

Step 8. Repeat for Trials 2 and 3. Average the data from the 3 trials and record in the Average Time column.

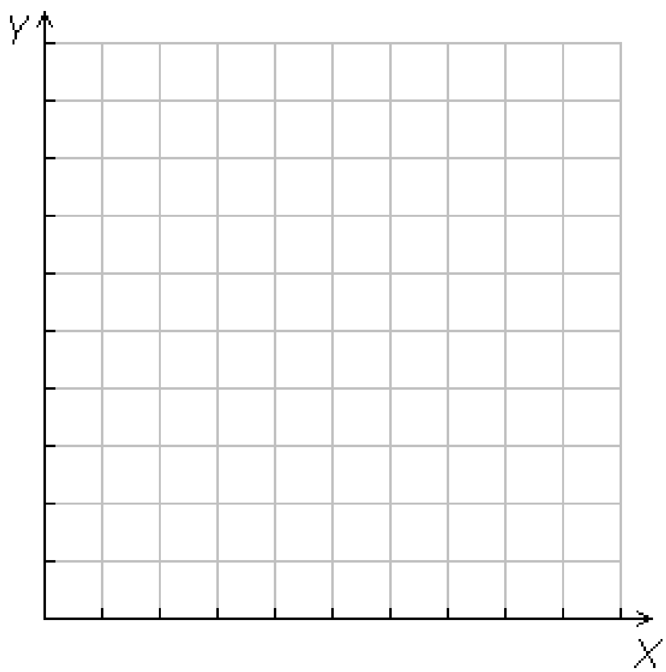
Step 9. The materials manager who is holding the pencil shortens the spring by wrapping it around the pencil until the desired length of 140 is obtained.

Step 10. Continue repeating the procedure with shortened lengths of rubber band spring. Continue to record your data.

1. Record your data in the table below.

Approximate Length of Spring (cm) *x*	Actual Length of Spring (cm) *x*	Trial 1	Trial 2	Trial 3	Average Time *y*
0					
20					
30					
40					
60					
80					
100					
120					
140					
160					

2. Make a scatterplot of the data you collected.



3. What is the independent variable?

4. What is the dependent variable?

5. Write a dependency statement relating the two variables.

6. What is a reasonable domain for the set of data?

7. What is a reasonable range for the set of data?

8. Is this data set continuous or discrete? Why?

9. Does the set of data represent a function? Why?

10. Write a summary statement about what happened in this data investigation.

11. Is the function increasing or decreasing?

12. Is the rate of change constant?

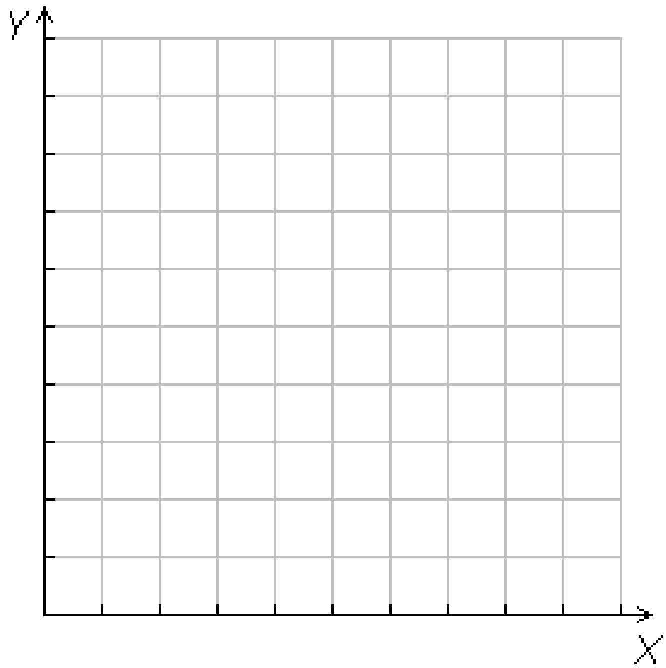
13. Does the data you collected appear to be a linear, quadratic, exponential, or some other type of parent function? Why?

14. How is the bottle bounce activity similar to the ticks of a metronome?

15. What kind of function do you think models the ticking of the metronome? Why?

<u>Part 3</u>: Analyzing the Data

1. How can you determine whether this function is the inverse of another parent function using patty paper?



2. Input your values into L_1 and L_2 of a graphing calculator, letting L_1 be independent values and L_2 dependent values, and create a scatterplot of the original data. Sketch your graph.

3. Create a second scatterplot that represents an inverse of the data. Use a different plot symbol for this scatterplot. Determine a new domain and range, and set a new viewing window. Sketch your graph.

4. What changes must you make to the window to view the second set of data?

5. Which parent function do the *reflected* points most closely appear to represent?

6. How did you determine your function?

7. How might your confirm your conjecture?

8. Without using regression, find a function that approximates your data for Plot 2.

9. Does your viewing window allow you to see both sides of the parabola? If not, readjust your viewing window. Sketch your graph.

10. How could you use this function to find a function that would approximate the first scatterplot you graphed?

11. Reset your window to view Plot 1. Enter the equation in the equation editor. Is your graph a close fit to the data in Plot 1? Sketch your graph.

12. Compare and contrast the graphs of a quadratic function and a square root function. How are they similar, and how are they different?

13. Why are there no negative coordinates in the square root function?

14. What is the domain of a square root function?

15. What is the range of a square root function?

16. What conclusions can you make about the attributes of a square root function?

17. What conclusions can you make about the collected data?

Part 4: Making Symbolic Generalizations

Transformations of Square Root Functions Card Sort

1. Place the cards in the proper row and column.

Description	Example	Example	Notation
Vertical Translation Up			
Vertical Translation Down			
Horizontal Translation Left			
Horizontal Translation Right			
Vertical Stretch			
Vertical Compression			
Reflection			

2. Describe the role of *a*.

3. Describe the role of *h*.

4. Describe the role of *k*.

5. Using *x, a, h*, and *k*, write an equation that could be used to summarize the transformations to the square root function.

6. Revisiting the bottle bounce investigation, describe the transformation to the square root parent function that represents your data.

<u>Part 5 (Optional Extension)</u>: Investigating the Coefficient of *x*.

1. Using $f(x) = a\sqrt{\frac{1}{b}(x-h)} + k$, predict the changes in the parent function for the following functions. Then check with your graphing calculator.

   a. $f(g) = \sqrt{-x}$

   b. $f(g) = \sqrt{-3x} + 4$

   c. $f(g) = \sqrt{\frac{1}{2}(x-3)}$

   d. $f(g) = 2\sqrt{-\frac{1}{3}(x+4)} - 5$

2. What can you summarize about transformations of the square root parent function as a result of changes to $\frac{1}{b}$?

<u>Part 6 (Optional Extionsion)</u>: Connecting the Roles of *a, h,* and *k* in Square Root and Quadratic Functions

Equation 1 $$y = 3\sqrt{x-5}+6$$

Equation 2 $$y = 7(x-2)^2+4$$

1. Find the inverse of Equation 1.

2. Numerically and graphically compare and contrast Equation 1 and its inverse.

3. Find the inverse of Equation 2.

4. Numerically and graphically compare and contrast Equation 2 and its inverse.

5. Find the inverse of $y = a\sqrt{(x-h)} + k$.

6. Find the inverse of $y = a(x-h)^2 + k$.

7. Summarize the relationship between h and k in the square root transformation form and h and k in the quadratic transformation (vertex) form.

8. Summarize the relationship between a in the square root transformation form and a in the quadratic transformation (vertex) form.

<u>Part 7</u>: Solving Square Root Equations and Inequalities

1. Consider the system of equations $y = \sqrt{x+3}$ and $y = 4$.
   a) Graph the system and sketch the graph. What are the domain and range for each function in this system?

   b) How can you determine the solution to this system of equations graphically or tabularly?

   c) What are the coordinates of the point that is a solution for this system?

   d) How can you use the transitive property to write this system as one equation?

   e) How can you solve this equation algebraically?

   f) What would be your solution set if you had been given $\sqrt{x+3} \geq 4$?

2. Consider the system of equations $y = \sqrt{x+2}$ and $y = x$.

a) Graph the system and sketch the graph. What are the domain and range for each function in this system?

b) How can you determine the solution to this system of equations graphically or tabularly?

c) What are the coordinates of the point that is a solution for this system?

d) How can you use the transitive property to write this system as one equation?

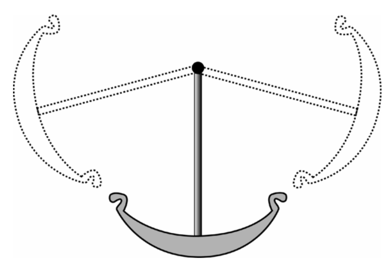
e) How can you solve this equation algebraically?

f) Are both solutions valid? Why or why not?

3. Choose one of the following problems. Work with your group to find the solutions(s). Justify your answer. Use chart paper to display your work.

   A. Jim is an accident investigator who was asked to determine whether a driver's excessive speed was a factor in a traffic accident. The traditional equation used to determine the speed at which a vehicle was traveling at the onset of the skid is $V_s = \sqrt{2aS_s}$, where a is the deceleration force of gravity times friction, and S_s is the length of the skid marks. If the speed limit is 60 mph and the skid marks are 225 ft. long, was the driver exceeding the speed limit? (Use 6.01 for a.) What is the maximum length skid mark that would have exonerated the driver? Justify your answer.

   B. At Thalia's favorite amusement park, there is a ride called the "Pirate Ship". People sit in what looks like a huge ship. The "ship" then swings back and forth. Thalia notices that it takes somewhere between 7 and 8 seconds for the ride to make one complete swing back and forth. What is the minimum and maximum length of the swinging bar?



The function that represents the time in seconds of one complete swing, t, based on the height of the swinging bar, h, in feet, is $t = 2\pi\sqrt{\frac{h}{32}}$.

C. Arnie was taking a picture from the window of his apartment. Unfortunately, he dropped the camera, which landed on the ground at least 2 seconds later. The equation that models the time, t, it takes for an object to fall h meters is $t = \sqrt{\frac{2h}{9.81}}$. From what height did Arnie drop the camera?

D. Sharon's mother bought a grandfather clock and asked Sharon to determine how long the pendulum must be so the clock keeps accurate time. Sharon found the formula $t = 2\pi\sqrt{\frac{L}{g}}$, where t is the time for one complete swing of the clock pendulum, L is the length of the pendulum, and g is acceleration due to gravity (which is 980 cm/sec^2). Since the time for a complete swing of the pendulum of a grandfather clock must be 2 seconds, how long should the pendulum be?