Leader Notes: How Much Will It Bend?

Purpose:

The purpose of this section of the professional development is to investigate stumbling blocks and challenges that students and teachers have in developing concepts and procedures involved with learning about rational functions.

Descriptor:

Explore phase 4 has four parts. The first part is an investigation of inverse variation by measuring deflection in various size bundles of linguine cantilevers. The second part looks at transformations to the reciprocal function $f(x) = \frac{1}{x}$. The third part investigates the rational function definition and connections between rational function form, factored form, transformation form, asymptotes of rational functions, and graphs of rational functions. The fourth part explores solving rational equations in a real work context.

Duration:

2.5 hours

TEKS:

a5 Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

a6 Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

2A.10 **Rational functions**. The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

2A.10A The student is expected to use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior.

2A.10B The student is expected to analyze various representations of rational functions with respect to problem situations.

2A.10C The student is expected to determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities.

2A.10D The student is expected to determine the solutions of rational equations using graphs, tables, and algebraic methods.

2A.10E The student is expected to determine solutions of rational inequalities using graphs and tables.

2A.10F The student is expected to analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic functions, and solve the problem.

2A.10G The student is expected to use functions to model and make predictions in problem situations involving direct and inverse variation.

TAKS™ Objectives Supported:

While the Algebra II TEKS are not tested on TAKS, the concepts addressed in this lesson reinforce the understanding of the following objectives.

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 10: Mathematical Processes and Mathematical Tools

Materials:

Prepare in Advance: Copies of participant pages, cut out cards for the **Rational Function Card Sort**, cut out cards for the **Rational Function Card Match**

Presenter Materials: Overhead graphing calculator

Per group: String, ruler, masking tape, clear packing tape, linguine, 35mm film canister or candy container that is a cylinder, pennies (approximately 5), deflection grid (optional)

Per participant: Copy of participant pages, graphing calculator

Explore

<u>Part 1</u>: Linguine Cantilever

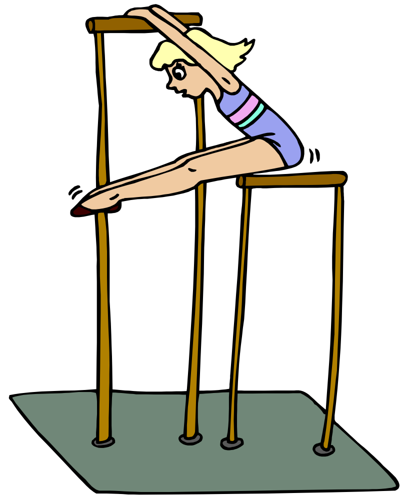
Leader Notes:
The purpose of Part 1 is to have participants actively involved in collecting data related to the reciprocal parent function. At the end of Part 1 the participants will be able to see how collecting data reinforces the understanding of the concept of the reciprocal parent function.

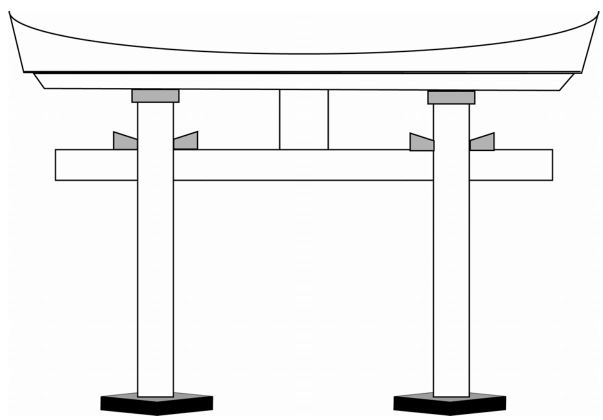
Ask participants to discuss how the objects on the first page are alike and how they are different. The idea you would like to elicit from the participants is that some of the objects are supported on two or more places of a beam and others are only supported at one end. Then proceed to discuss the porches on Fallingwater that are supported at only one end.

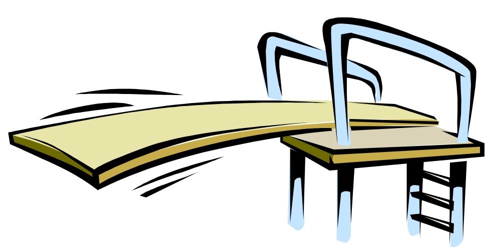
In this phase of the professional development participants investigate the reciprocal function generated by modeling the amount of deflection from a cantilevered beam. In this simplified experiment we are using linguine. It is not necessary that you use linguine, spaghetti or fettuccine will also work. If 35mm film canisters are not readily available, you can use pennies placed in a baggie, secured with transparent tape, and hung with an unfolded paper clip as your load. All participants should use the same type of linguine from a freshly opened package if you wish to average the data from the groups.

You may want to search for websites with information about cantilevers. For example, Wikipedia, the new observation deck for the Grand Canyon, or Fallingwater information.

How are the objects below alike? How are they different?





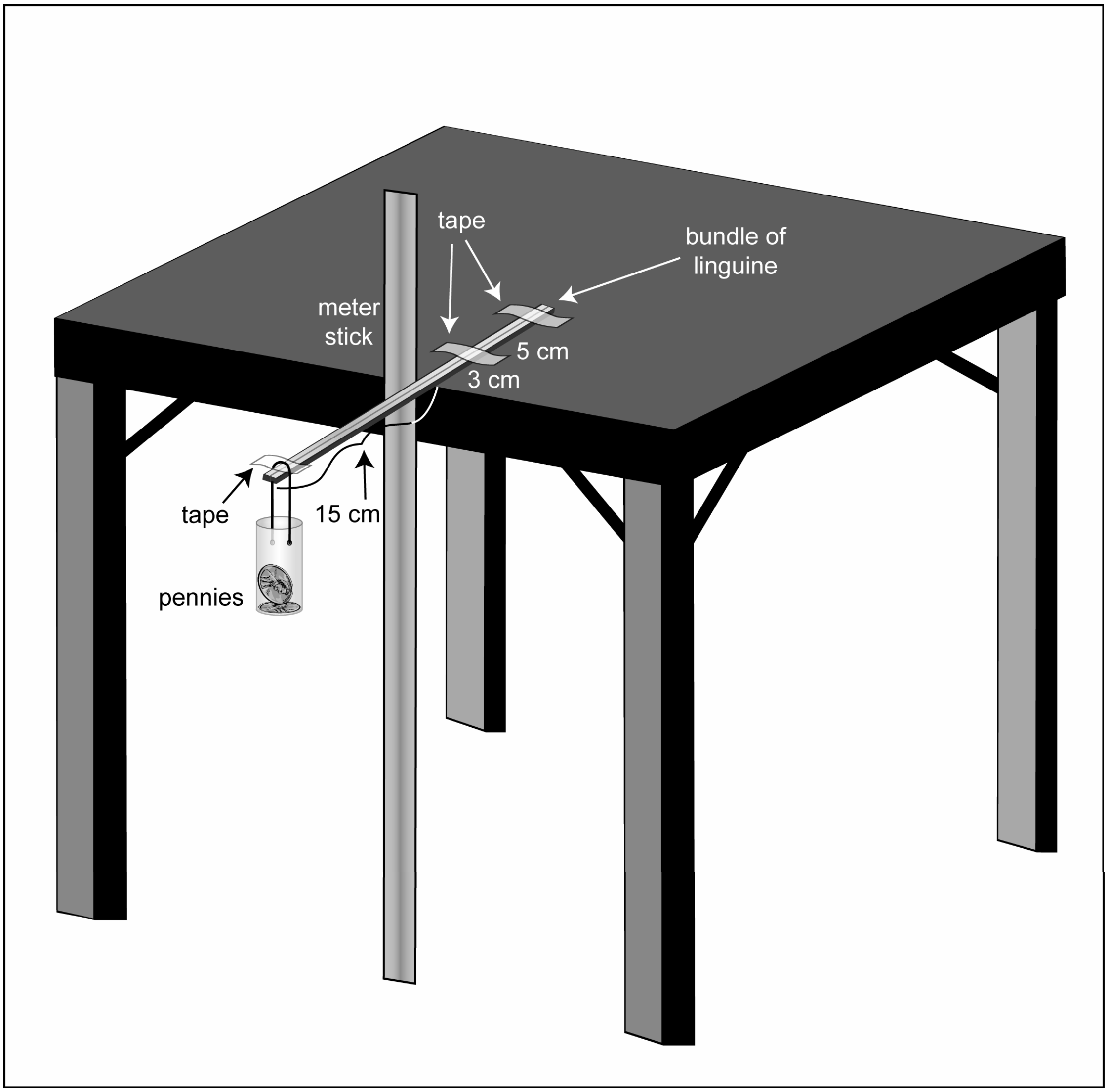


A cantilever is a projecting structure that is secured at only one end and carries a load on the other end. Diving boards and airplane wings are examples of horizontal cantilevers. Flagpoles and chimneys are vertical cantilevers. One of the most famous examples of a cantilever in architecture, which is shown below, is the Frank Lloyd Wright designed home, Fallingwater. The strength of a cantilever can be affected by variables such as length, load, cross sectional area, temperature, or elasticity. In this activity, you will be investigating the relationship between the thickness of a cantilever and the deflection in the cantilever when weight is added at the end.

In this investigation, you will keep the length of a piece of linguine that is hanging over the edge of a desk constant as you collect data on how much the linguine deflects. Deflection is the measure of the amount that the linguine bends in the downward direction. The number of pieces of linguine will change. Since you want to keep all variables (except for the ones you are investigating) constant, make sure to pay attention to the hints listed with the instructions.

Divide into groups of 3. Each person in the group has a job.

Materials manager:	Get the necessary materials, direct the team in setting up the investigation
Measures manager:	Measure the amount of deflection as the investigation proceeds
Data manager:	Record the necessary measurements in the table, share the data with the team



Data Collection Set-up Instructions

Step 1. **The materials person should get the necessary materials and begin to make bundles of linguine. Each bundle of 1, 2, 3, 4, 5, 6, 7, and 8 pieces of linguine should be taped one inch from each end. Since linguine is not all exactly the same length, try to keep one end of the bundle lined up.**

Step 2. **Tape a short piece of string to the 35mm film canister to form a handle. If you do not have a film canister, use a baggie, transparent tape, and a paper clip to build a weight to hang from the linguine.**

Step 3. **Tape one piece of linguine with 15 centimeters hanging over the edge of a desk. Put one piece of tape approximately 3 cm from the edge of the table. Place a second piece of tape over the end of linguine. Place the load (film canister) on the end of the linguine that is hanging over the edge of the desk. Slowly place pennies in the film canister until the linguine breaks. Wait 15 seconds before adding an additional penny. Use one less penny than the number required to break one piece of linguine as the load in your bucket for the remainder of this data collection experiment.**

Step 4. **Tape a meter stick perpendicularly to the floor next to a desk.**

Step 5. **Measure the linguine's height above the floor without the film canister attached. (Hint: It is easier to consistently measure the height using the bottom of the linguine.)**

Step 6. **Place your pennies into the bucket. (Hint: Place the pennies gently, throwing pennies into the bucket will alter the results.)**

Step 7. **Place the bucket on the end of the linguine that is hanging over the edge of the desk. (Hint: Place the string at the same point on the linguine for each trial. Use a piece of masking tape to hold the bucket onto the linguine.)**

Step 8. **Wait 15 seconds. Measure the amount of deflection in the linguine. (Hint: An easy method for measuring deflection is to use the eraser end of a pencil to line up the deflection of the end of the linguine with its measure on the meter stick.) Record your measurements in the table.**

Step 9. **Repeat the procedure with two pieces of linguine taped together still hanging 15 centimeters over the edge of the desk. Measure the deflection of the bundle of linguine.**

Step 10. **Continue repeating the procedure with additional pieces of linguine until you measure deflection with eight pieces taped together. Continue to record your data.**

1. **Fill in the table with the data you collected.**

Sample response:

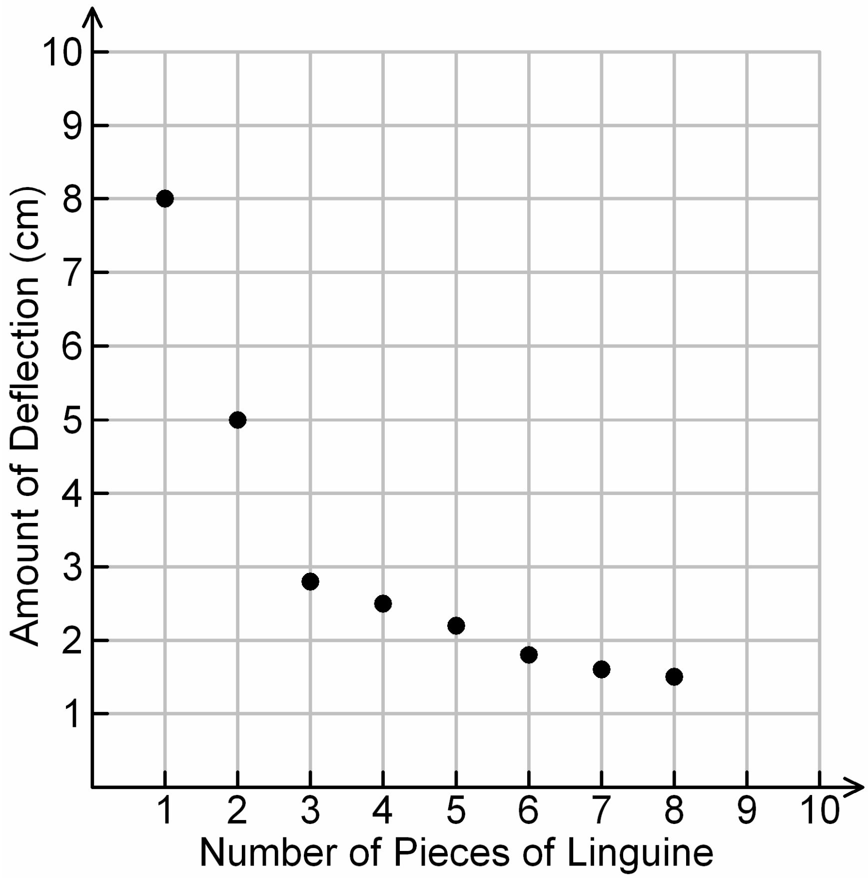
Number of Pieces of Linguine in the Bundle (x)	**Starting Height of Linguine Bundle Above the Floor**	**Height of Linguine Bundle Above the Floor After the Load is Placed**	**Amount of Deflection in the Linguine (y)**	**Product of x and y ($x \cdot y$)**
1	*74 cm*	*66 cm*	*8 cm*	*8*
2	*74 cm*	*69 cm*	*5 cm*	*10*
3	*74 cm*	*71.2 cm*	*2.8 cm*	*8.4*
4	*74 cm*	*71.5 cm*	*2.5 cm*	*10*
5	*74 cm*	*71.8 cm*	*2.2 cm*	*11*
6	*74 cm*	*72.2 cm*	*1.8 cm*	*10.8*
7	*74 cm*	*72.4 cm*	*1.6 cm*	*11.2*
8	*74 cm*	*72.5 cm*	*1.5 cm*	*12*

2. **Write a dependency statement relating the two variables.**
The deflection of the linguine bundles depends on the number of pieces of linguine in the bundle.

3. **What is a reasonable domain for the set of data?**
Responses may vary. The number of pieces of linguine might vary from 1 to 10.

4. **What is a reasonable range for the set of data?**
Responses may vary. The deflection appears to be between 1 and 9.

5. Make a scatterplot of the data you collected.



6. Verbally describe what happens in this data collection investigation.
As the number of pieces of linguine increases the deflection gets smaller.

7. Is this data set continuous or discrete? Why?
The data set is discontinuous; it is very difficult to use a fraction of a piece of linguine.

8. Does the set of data represent a function? Why?
Yes, the data set represents a function. For each bundle of linguine, there is one measured deflection value.

9. Does the data appear to be a linear, quadratic, exponential or some other type of parent function? Why do you think so?
The data does not appear to be linear or quadratic. It might appear to be exponential. If you investigate successive quotients, they are not congruent. So, this appears to be a new type of parent function.

10. Is the function increasing or decreasing?

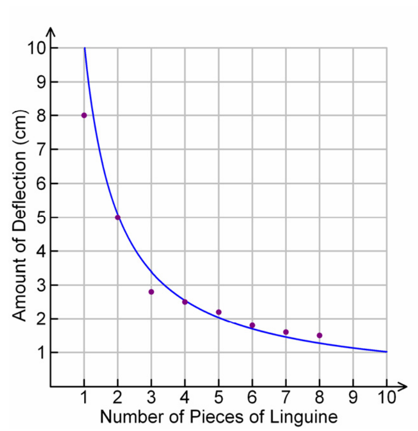
The function values are decreasing, which makes sense since the more linguine in the bundle, the less deflection there appears to be.

11. Is the rate of change constant for this set of data?

The rate of change is not constant for this set of data. The rate of change varies depending on which pairs of points you investigate.

12. Determine a function rule that models the set of data you collected.

If you find the average of the products of x and y, you will have a close approximation to a in the function $y = \frac{a}{x}$. *Using the sample date, the function that models the data is approximately* $y = \frac{10.075}{x}$.



13. To get a better model add your set of data to the data of the entire group. Each group should send their data manager to the overhead to fill in the data collected for their group. Record the additional data in the table below. Find the average deflection for each bundle of linguine for the entire group.

Number of Pieces of Linguine in the Bundle	Amount of Deflection for Each Team												Average
	A	B	C	D	E	F	G	H	I	J	K	L	
1													
2													
3													
4													
5													
6													
7													
8													

14. Using the entire group's data, what function would you now use to model this situation?
Responses will vary. Hopefully, the data collected from the entire group will be a better approximation of the data.

15. How does this investigation connect to the TEKS from previous courses?
Responses may vary. Students should have investigated inverse variation in Algebra I. TEKS A.11B states, "The student is expected to analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods."

As a whole group discuss the answer to question 16 before moving on to Part 2.

16. What are the key points students need to understand about the Linguine Cantilever before continuing the investigation of rational functions?
Responses may vary. Students need to understand that the product of the two variables is a constant in an inverse variation situation. They should be able to distinguish the inverse variation situation from previous parent functions they have studied. The average of the entire class's data may be a better model for the situation than the data from one group. Students are remembering what they have learned previously about inverse variation.

Part 2. Transformations to $f(x)=\frac{1}{x}$

Leader notes:
The purpose of Part 2 is to emphasize the fact that the reciprocal parent function is related to the linear parent function. At the end of this part, participants should recognize the need to be more overt in demonstrating how the function values in the table change, in addition to changes to the graph during vertical dilations, vertical shifts, and horizontal shifts to the parent function. Explain to participants that they will be investigating a parent function new to Algebra 2 students.

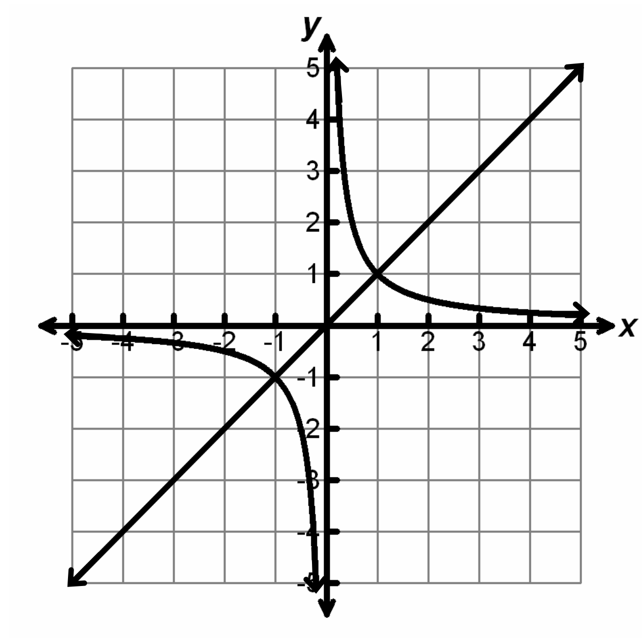
1. **What is the reciprocal of the linear parent function,** $f(x)=x$**?**

   The reciprocal is $g(x)=\frac{1}{x}$.

2. **Let's investigate some of the attributes of the function and its reciprocal. Fill in the tables with several values for each function. Draw a sketch of the graphs of the two functions on the same set of axes.**
   Sample response:

$f(x)=x$	
x	***y***
–3	*–3*
–2	*–2*
–1	*–1*
–0.5	*–0.5*
–0.1	*–0.1*
0	*0*
0.1	*0.1*
0.5	*0.5*
1	*1*
2	*2*
3	*3*

$f(x)=\frac{1}{x}$	
x	***y***
–3	*–1/3*
–2	*–1/2*
–1	*–1*
–0.5	*–2*
–0.1	*–10*
0	*0*
0.1	*10*
0.5	*2*
1	*1*
2	*1/2*
3	*1/3*



3. Using your graphing calculator (if necessary), fill in the tables below. Let $f(x) = x$ be Y_1, and let $g(x) = \frac{1}{x}$ be Y_2.

$$Y_1 = x$$

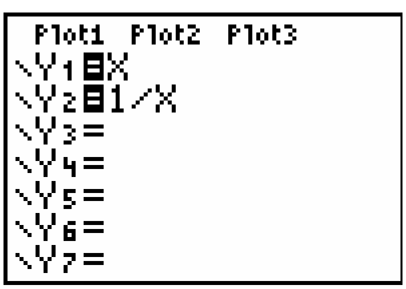
$$Y_2 = \frac{1}{x}$$

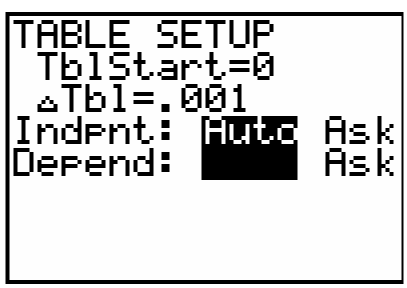
$Y_1 = x$		$Y_2 = \frac{1}{x}$
$(-\infty, \infty)$	**Intervals where the function is increasing**	*none*
none	**Intervals where the function is decreasing**	$(-\infty, 0) \cup (0, \infty)$
none	**Intervals where the function is undefined**	$x = 0$
$(0, 0)$	**Coordinates of the *x*-intercepts (zeros)**	*none*
none	**Equations of any asymptotes**	$x = 0$ $y = 0$

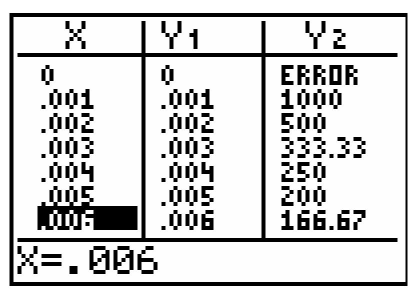
4. What do you notice about the graphs of the linear parent function and its reciprocal?
Whenever the linear function is increasing, the reciprocal is decreasing. The x-intercept of the linear function is one of the asymptotes of the reciprocal function.

5. Where do the linear parent function and its reciprocal intersect?
They intersect at (1, 1) and (–1, –1).

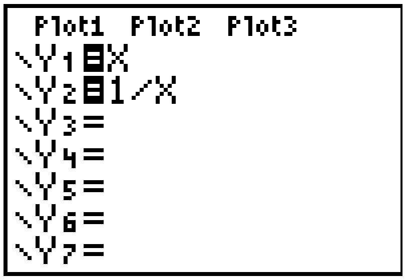
6. How could you have your students investigate what happens to *f(x)* as *x* gets closer and closer to 0 using the graphing calculator?
Responses may vary. Set the ΔTbl to a small number such as 0.001 and investigate the y values.

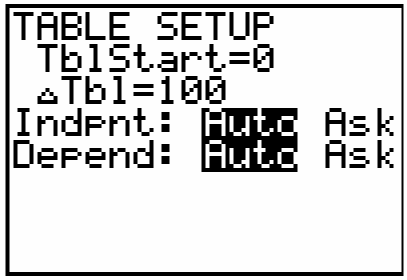


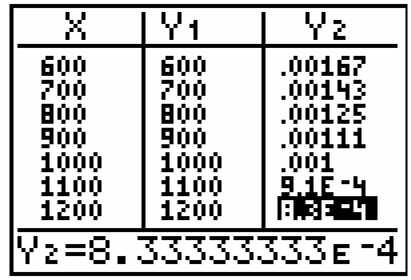




7. How could you have your students investigate what happens to *f(x)* as *x* gets larger and larger?
Actually look at the y values in the table as x gets larger. The y values get closer and closer to 0.







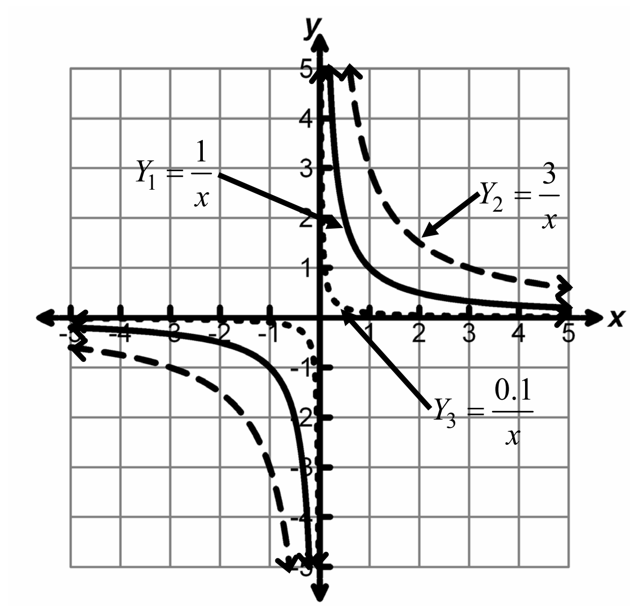
8. How do the Algebra II TEKS name this new parent function?
In the Algebra II TEKS this new parent function is the reciprocal function.

9. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x)=\frac{1}{x}$, when it is multiplied by a constant as in the examples below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results.

$Y_1=\frac{1}{x}$

$Y_2=\frac{3}{x}$

$Y_3=\frac{0.1}{x}$



Sample response:

x	$Y_1=\frac{1}{x}$	$Y_2=\frac{3}{x}$	$Y_3=\frac{0.1}{x}$
−3	−0.33…	−1	−0.03…
−2	−0.5	−1.5	−0.05
−1	−1 →	−3	−0.1
0	error	error	error
1	1 →	3 →	0.1
3	0.33…	1	0.033…
2	0.5	1.5	0.05
4	0.25	0.75	0.25

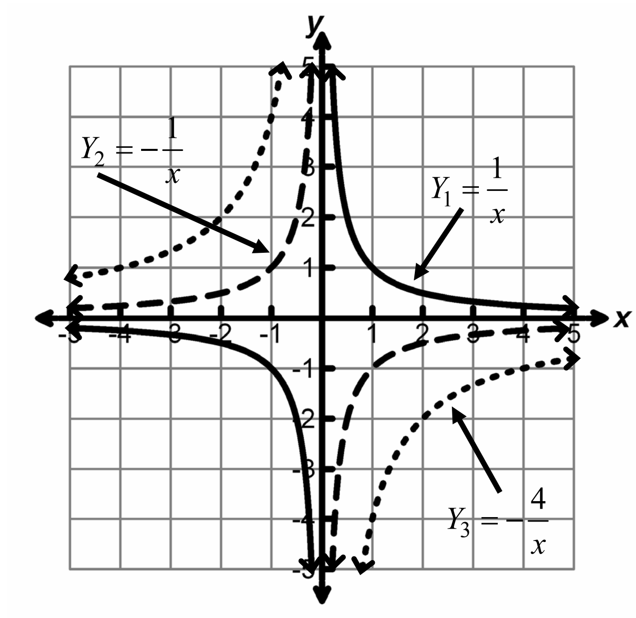
If the constant is larger than 1, the graph is stretched vertically. If the constant is less than one but greater than zero, the graph is vertically compressed.

10. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x)=\frac{1}{x}$, when it is multiplied by a negative constant as in the examples below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results which includes any asymptotes.

$Y_1=\frac{1}{x}$

$Y_2=-\frac{1}{x}$

$Y_3=-\frac{4}{x}$



x	$Y_1=\frac{1}{x}$	$Y_2=-\frac{1}{x}$	$Y_3=-\frac{4}{x}$
−3	−0.33…	0.33…	1.33…
−2	−0.5	0.5	2
−1	−1	1	4
0	error	error	error
1	1	−1	−4
2	0.5	−0.5	−2
3	0.33…	−0.33…	−1.33…
4	0.25	−0.25	−1

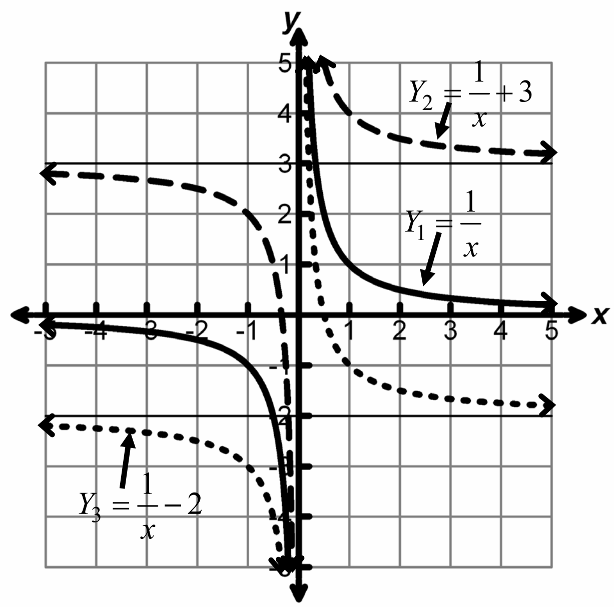
Sample response: The y-values in Y_2 and Y_3 are the opposite sign of the y-value in Y_1. So, it would appear that the graph of the parent function is reflected across the x-axis. All three functions still have "error" at x = 0, a vertical asymptote. The vertical stretching properties are the same. There is a horizontal asymptote at y = 0.

11. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x)=\frac{1}{x}$, if a constant is added to the function as in the three functions listed below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results which includes any asymptotes.

$Y_1=\frac{1}{x}$

$Y_2=\frac{1}{x}+3$

$Y_3=\frac{1}{x}-2$



x	$Y_1=\frac{1}{x}$	$Y_2=\frac{1}{x}+3$	$Y_3=\frac{1}{x}-2$
–3	–0.3…	2.6…	–2.3…
–2	–0.5	2.5	–2.5
−1	−1	2	−3
0	error	error	error
1	1	4	–1
2	0.5	3.5	−1.5
3	0.3…	3.3…	–1.6…
4	0.25	3.25	−1.75

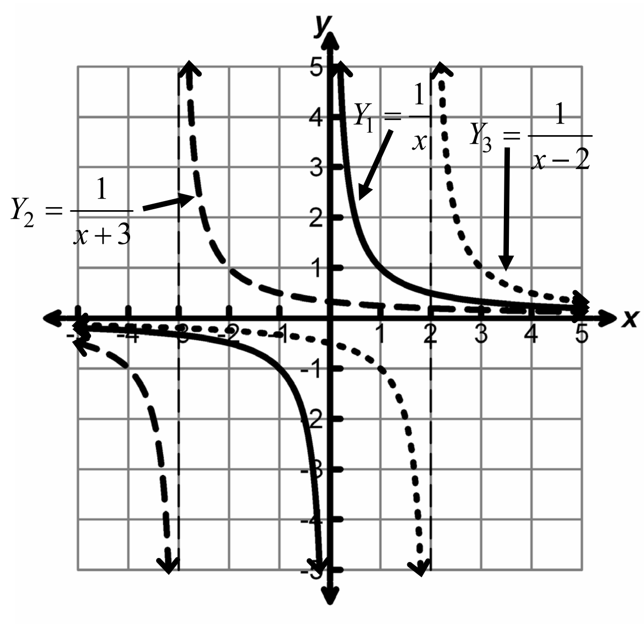
Sample response: Each y-value in Y_2 is greater than the corresponding Y_1 value by 3. Each y-value in Y_3 is 2 less than the corresponding y-value in Y_1. The graphs appear to slide up or down vertically. The horizontal asymptote is changing. The horizontal asymptote appears to occur at the value of the vertical shift. A vertical asymptote still occurs at $x = 0$.

12. Using your graphing calculator describe what happens to the reciprocal parent function, $g(x)=\frac{1}{x}$, if a constant is added to the *x*-coordinate in the denominator as in the three functions listed below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results which includes any asymptotes.

$Y_1=\frac{1}{x}$

$Y_2=\frac{1}{x+3}$

$Y_3=\frac{1}{x-2}$



x	$Y_1=\frac{1}{x}$	$Y_2=\frac{1}{x+3}$	$Y_3=\frac{1}{x-2}$
−3	−0.333…	error	−0.2
−2	−0.5	1	-0.25
−1	−1	0.5	−0.333…
0	error	0.333…	−0.5
1	1	0.25	−1
2	0.5	0.2	error
3	0.333…	0.166…	1
4	0.25	0.143	0.5

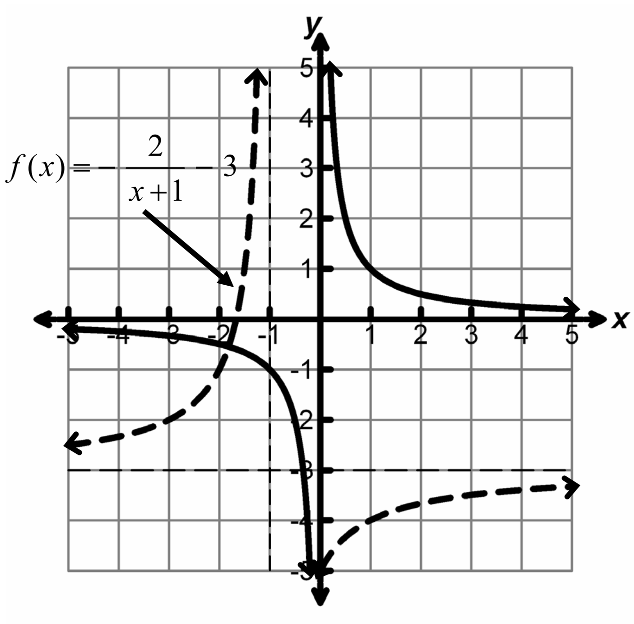
Possible response: The graphs appear to have been shifted horizontally left and right depending on the sign of the constant. Looking at the table, the y-values shift 3 units to the left or 2 units to the right, depending on the function. The horizontal asymptote for each of the functions is y = 0. The vertical asymptotes are changing depending on the value of the constant added to x.

13. Predict, describe and then sketch the transformations to the reciprocal parent function in the function below.

$$f(x)=-\frac{2}{x+1}-3$$

Possible response: The function will be reflected over the x-axis, stretched vertically by a factor of 2, slid to the left one unit, and slid down three units. The horizontal asymptote is y = −3, and the vertical asymptote is x = −1.

x	$Y_1 = \frac{1}{x}$	$Y_1 = -\frac{2}{x}$	$Y_1 = -\frac{2}{x+1}$	$Y_2 = -\frac{2}{x+1} - 3$
−3	*−0.333…*	*0.66…*	*1*	*−2*
−2	*−0.5*	*1*	*2*	*−1*
−1	*−1*	*2*	*error*	*error*
0	*error*	*error*	*−2*	*−5*
1	*1*	*−2*	*−1*	*−4*
2	*0.5*	*−1*	*−0.66…*	*−3.66…*
3	*0.333…*	*−0.66…*	*−0.5*	*−3.5*
4	*0.25*	*−0.5*	*−0.4*	*−3.33…*



14. Using the variables *a*, *h*, and *k* describe how transformations to the reciprocal function are similar to transformations to other parent functions.
Transformations to parent functions have the same properties. This helps students connect their new knowledge to previous concepts.

As a whole group discuss the answer to question 15 before moving on to Part 3.

15. What are the key points students need to understand about transformations to the reciprocal function before continuing the investigation of rational functions?
Responses may vary. Students need to understand that the reciprocal parent function is affected by transformations in the same way as the quadratic parent function. The transformations can be justified using a graph, an equation, or a table.

Part 3: Rational Functions

Leader notes:
The purpose of Part 3 is to have participants clarify their understanding of the definition of a rational function.

1. **Give each pair of participants a set of the cards, *Rational Function Card Sort*. Have them sort the functions into two (and only two) groups. Try not to give any hints ahead of time that some of the functions are rational functions and some are not. After all of the groups have sorted their cards, encourage them to describe their sorting method.**

   The rational functions are b(x), d(x), h(x), j(x), k(x), n(x), p(x), q(x), t(x), w(x), y(x), z(x). Functions that are not rational are c(x), f(x), g(x), m(x), r(x), s(x), u(x), and v(x).

Facilitation Questions

- **What is the definition of a rational function?**
  A rational function is any expression that can be written in the form $\frac{p(x)}{q(x)}$ *where p(x) and q(x) are both polynomials and q(x) ≠ 0.*

- **What is the definition of a polynomial?**
  A polynomial in one variable is any expression that can be written in the form
  $$a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x^1 + a_0$$
  where x is a variable, the exponents are nonnegative integers, and the coefficients are real numbers. (Discovering Advanced Algebra, p. 360)
  Some definitions allow coefficients to be complex numbers.
  "Polynomials have no variables in denominators or exponents, no roots or absolute values of variables, and all variables have whole number exponents." (Holt, Algebra 2)

- **How does what students have learned previously about rational numbers relate to their new knowledge of rational functions?**
  Students learned about fractions (rational numbers) in elementary and middle school. They learned about inverse variation in Algebra 1. Now, the reciprocal parent function is introduced as a subset of the larger set of rational functions.

- **What are the most interesting parts of rational functions?**
  Responses may vary. A possible response might be: "The most interesting parts are the vertical and horizontal asymptotes and holes."

- **Which polynomials are Algebra II students required to use in rational functions based on the Algebra II TEKS?**
  The TEKS require only that students use polynomials of degree 1 or 2 in the numerator and denominator of a rational function.

2. **After participants have completed the Rational Function Card Sort as a whole group discuss any differences in their sorting methods before proceeding.**

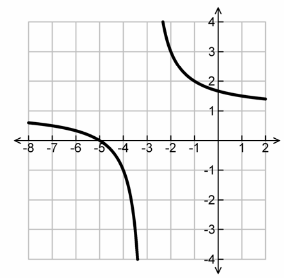
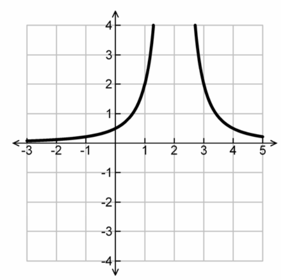
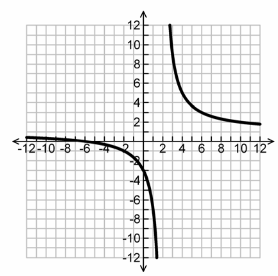
3. **Give each pair of participants a set of the cards, Rational Function Card Match. Have participants match the rational form of each rational function to the transformation form or factored form, the equations of any asymptotes, the coordinates of any removable discontinuities (holes), and the graph of the function.**

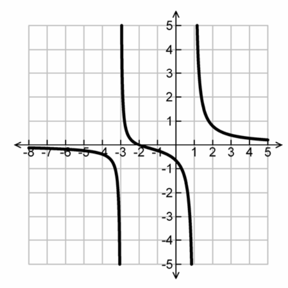
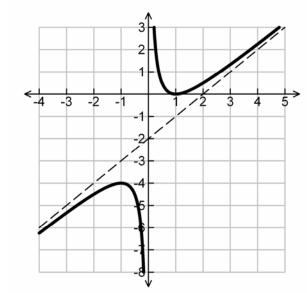
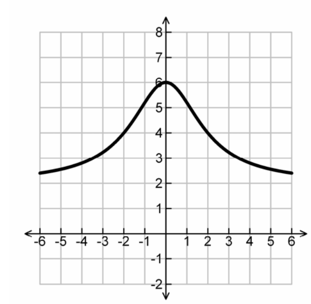
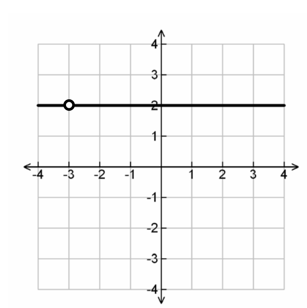
After participants have completed the Rational Function Card Match as a whole group discuss the answer to question 4 before moving on to Part 4.

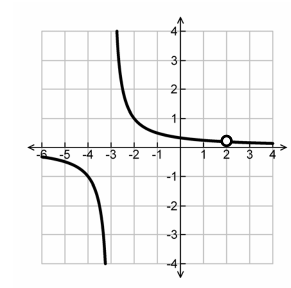
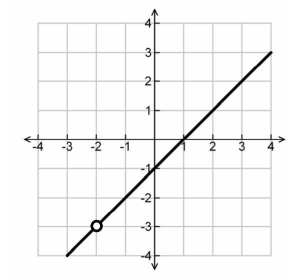
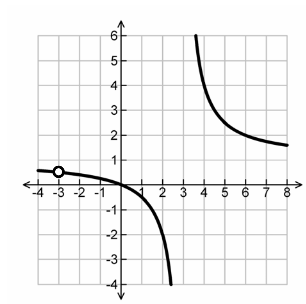
4. **What are the key points students need to understand about rational functions to be able to do the Rational Function Card Match?**
   Responses may vary. Students need to understand that rational functions can be written in several forms. One form might be good for finding the vertical asymptotes or x-intercepts. Another form might be good for finding an oblique asymptote. Students should be able to match the graph to its equations and identify any asymptotes or holes.

Rational Function Card Match

Answer Key

Rational Function Form	Transformation Form	Discontinuities, Asymptotes, Other Noteworthy Points	Graph
$f(x)=\dfrac{x+5}{x+3}$ **A**	$f(x)=\dfrac{2}{x+3}+1$	$x=-3$ $y=1$ x-intercept at (−5, 0) y-intercept at $(0, \frac{5}{3})$	
$f(x)=\dfrac{2}{x^2-4x+4}$ **B**	$f(x)=\dfrac{2}{(x-2)^2}$	$x=2$ $y=0$ no x-intercepts y-intercept at (0, 0.5)	
$f(x)=\dfrac{x+6}{x-2}$ **C**	$f(x)=\dfrac{8}{x-2}+1$	$x=2$ $y=1$ x-intercept at (−6, 0) y-intercept at (0, −3)	

$f(x)=\dfrac{x+2}{x^2+2x-3}$ **D**	$f(x)=\dfrac{x+2}{(x+3)(x-1)}$	$x=1$ and $x=-3$ $y=0$ x-intercept at $(-2, 0)$ y-intercept at $(0, -\frac{2}{3})$	
$f(x)=\dfrac{x^2-2x+1}{x}$ **E**	$f(x)=x-2+\dfrac{1}{x}$	$y=x-2$ $x=0$ x-intercept at $(1, 0)$ no y-intercepts	
$f(x)=\dfrac{2x^2+24}{x^2+4}$ **F**	$f(x)=\dfrac{16}{x^2+4}+2$	$y=2$ no vertical asymptotes no x-intercepts y-intercept at $(0, 6)$	
$f(x)=\dfrac{2x+6}{x+3}$ **G**	$f(x)=2,$ $x\neq-3$	no horizontal asymptote removable discontinuity (hole) at $(-3, 2)$ no x-intercepts y-intercept at $(0, 2)$	

$f(x)=\dfrac{x-2}{x^2+x-6}$ **H**	$f(x)=\dfrac{1}{x+3}$, $x\neq -2$	$x = -3$ and $y = 0$ removable discontinuity (hole) at (2, 0.2) no x-intercepts y-intercept at (0, $\frac{1}{3}$)	
$f(x)=\dfrac{x^2+x-2}{x+2}$ **I**	$f(x)=x-1$, $x\neq -2$	no horizontal asymptote removable discontinuity (hole) at (−2, −3) x-intercept at (1, 0) y-intercept at (0, −1)	
$f(x)=\dfrac{x^2+3x}{x^2-9}$ **J**	$f(x)=\dfrac{3}{x-3}+1$, $x\neq -3$	$x = 3$ and $y = 1$ removable discontinuity (hole) at (−3, 0.5) x-intercept at (0, 0) y-intercept at (0, 0)	

Part 4: Length of a Yellow Light

Leader notes:
The purpose of Part 4 is to allow participants to investigate a rational function in a real world context. At the end of the investigation they should be more willing to consider solving rational equations with tables and graphs in addition to the traditional symbolic solution.

One of the formulas traffic engineers use to help them calculate the length of time a traffic light should remain yellow is

$$Y(t) = t + \frac{v}{2a} + \frac{w+L}{v}.$$

The formula takes into account reaction time, braking time, and intersection clearance time. The variables used in this calculation are:

- t = reaction time (usually 1 second)
- v = velocity of the vehicle (in feet/second)
- a = deceleration rate (approximately 10 feet/ sec^2)
- w = width of the intersection (feet)
- L = length of the vehicle (feet)

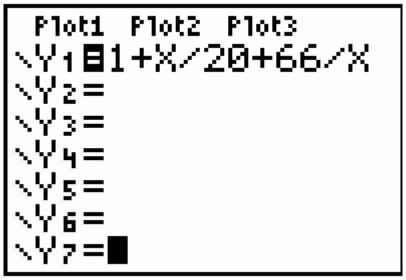
In order to calculate the speed limit for a certain intersection that is 48 feet wide, the engineer uses an average car length of 18 feet. She can calculate the length of time the traffic signal should remain yellow at that intersection based on the velocity in feet/second using the following formula:

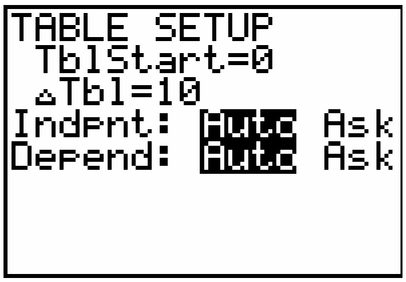
$$Y(v) = 1 + \frac{v}{20} + \frac{66}{v}.$$

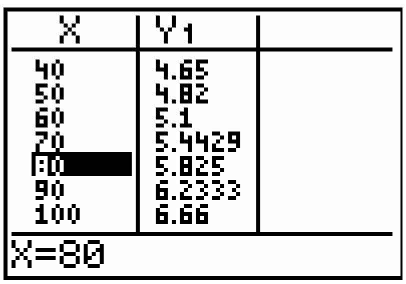
1. **If the posted speed limit at the intersection is 55 miles per hour, how long should the signal remain yellow?**
   Change the 55 mph speed limit into feet per second:

$$\frac{55\text{ miles}}{1\text{ hour}} \cdot \frac{1\text{ hour}}{3600\text{ seconds}} \cdot \frac{5280\text{ feet}}{1\text{ mile}} = \frac{80.667\text{ feet}}{1\text{ second}}.$$

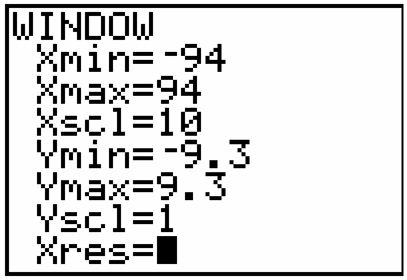
Solution using a table:

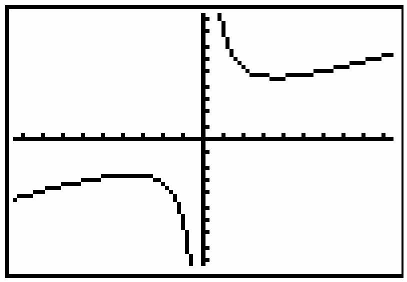


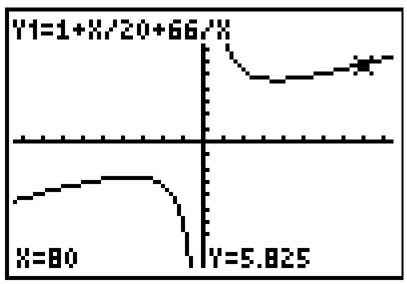




Solution using a graph:







Symbolic solution:

$$Y(v) = 1 + \frac{80.7}{20} + \frac{66}{80.7},$$

find $Y(v)$ when $v = 80.7$ ft/sec

so, $Y(80.7) \approx 5.85$ sec

So, if the posted speed limit is 55 mph, then according to the formula, the traffic signal should remain yellow for approximately 6 seconds.

2. **For a traffic signal to remain yellow for 4 seconds, what should the department of transportation post as the speed limit?**
   According to the formula applied to this particular intersection, the traffic signal should never remain yellow for only 4 seconds.

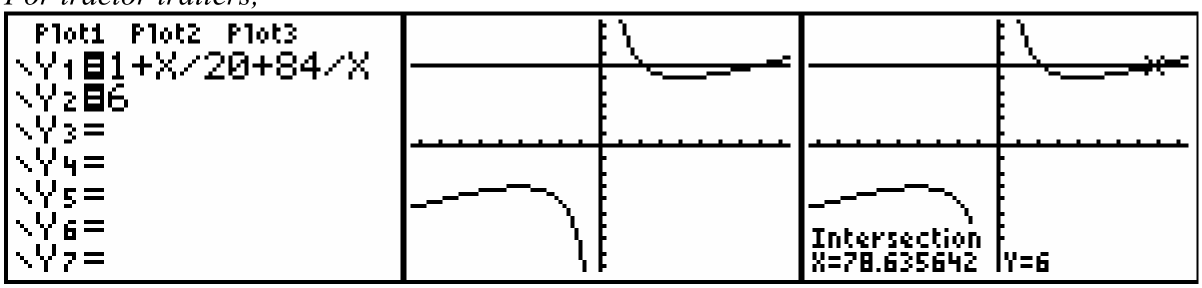
3. **If the speed of vehicles at a particular intersection varies between 30 and 50 mph, how long do you think the traffic signal should remain yellow?**
   First change 30 mph to 20.45 ft/sec and 50 mph to 34.09 ft/sec. Looking at the table and the graph at 30 mph the light should remain yellow for 5.2 seconds and at 50 mph the light should remain yellow for 4.6 seconds. The best time for the signal to remain yellow might be 5.5 seconds.

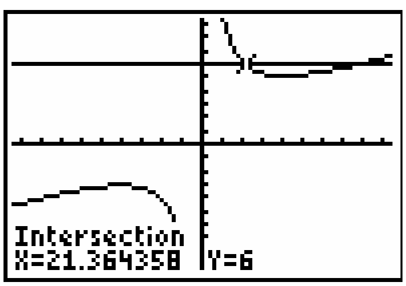
4. A tractor trailer that is approximately 36 feet long travels through the same intersection when the signal remains yellow for 6 seconds. Based on the formula, how fast should the tractor trailer be allowed to drive through the intersection? How fast should a car be allowed to drive through the intersection?

$$Y(v) = 1 + \frac{v}{20} + \frac{84}{v}$$

The speed limit for tractor trailers at one time should be 78.64 ft/sec, or approximately 53 mph; the second time the signal remains yellow for 6 seconds is 21.36 ft/sec, or approximately 15 mph. The speed limit for cars at one time should be approximately 84.35 ft/sec, or approximately 57.5 mph; the second time the signal remains yellow for 6 seconds is 15.65 ft/sec, or 10.7 mph.

For tractor trailers,





Using the table feature,

X	Y1	
74	5.8351	
75	5.87	
76	5.9053	
77	5.9409	
78	5.9769	
79	6.0133	
80	6.05	

X=78

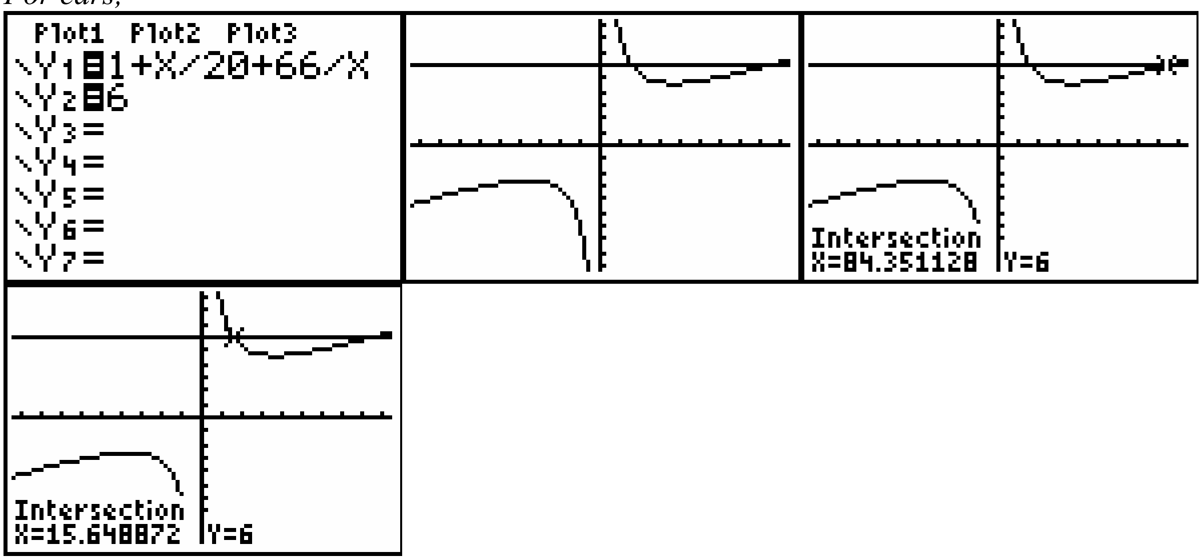
X	Y1	Y2
17	6.7912	6
18	6.5667	6
19	6.3711	6
20	6.2	6
21	6.05	6
22	5.9182	6
23	5.8022	6

X=20

Symbolically,

$$
\begin{aligned}
6 &= 1+\frac{v}{20}+\frac{84}{v} \\
20v(6) &= \left(1+\frac{v}{20}+\frac{84}{v}\right)20v \\
120v &= 20v+v^2+1680 \\
0 &= v^2-100v+1680 \\
v &= \frac{100 \pm \sqrt{(-100)^2-4(1680)}}{2} \\
v &\approx 78.64 \text{ ft/sec or } v \approx 21.36 \text{ ft/sec} \\
v &\approx 53 \text{ mph or } v \approx 15 \text{ mph}
\end{aligned}
$$

For cars,



Using the table feature,

X	Y1	
81	5.8648	
82	5.9049	
83	5.9452	
84	5.9857	
85	6.0265	
86	6.0674	
87	6.1086	

X=84

X	Y1	Y2
13	6.7269	6
14	6.4143	6
15	6.15	6
16	5.925	6
17	5.7324	6
18	5.5667	6
19	5.4237	6

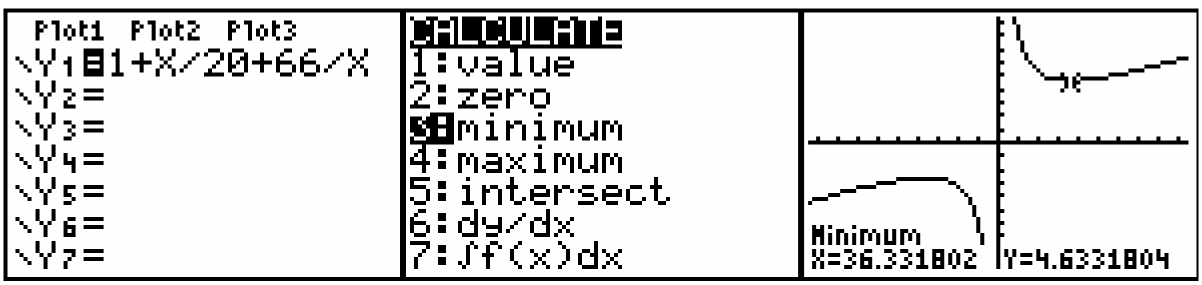
X=16

Symbolically,

$$
\begin{aligned}
6 &= 1+\frac{v}{20}+\frac{66}{v} \\
20v(6) &= \left(1+\frac{v}{20}+\frac{66}{v}\right)20v \\
120v &= 20v+v^2+1320 \\
0 &= v^2-100v+1320 \\
v &= \frac{100\pm\sqrt{(-100)^2-4(1320)}}{2} \\
v &\approx 84.35 \text{ ft/sec or } v\approx 15.65 \text{ ft/sec} \\
v &\approx 57.5 \text{ mph or } v\approx 10.7 \text{ mph}
\end{aligned}
$$

It is probably safer for the tractor trailers to travel approximately 5 miles per hour slower than the cars.

5. **Do you think this is a good model for length of time that traffic signals should remain yellow for every *x* value in the domain?**
   Responses may vary. Some participants may think that the model only seems to work well for speeds larger than 36 ft/sec, or approximately 25 mph, at this particular intersection. The rational function approaches an asymptote at x = 0, so for extremely slow speeds the length of the yellow light is very long.



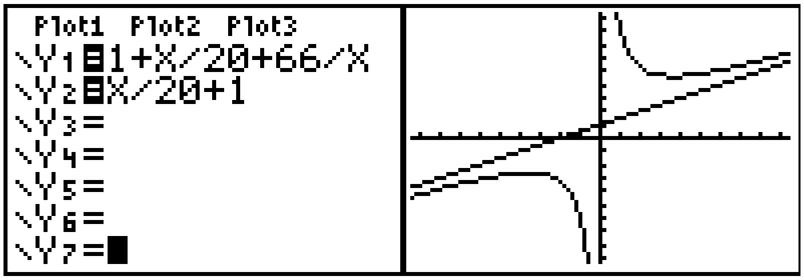
Bonus question:

6. **What is the oblique asymptote for this rational function? Graph both the function and the asymptote on your graphing calculator.**

$$Y(v)=1+\frac{v}{20}+\frac{66}{v}$$

As v gets larger and larger, $\frac{66}{v}$ *approaches 0. So, the equation of the oblique asymptote is*

$Y(v)=1+\frac{v}{20}.$



After participants have completed the Length of a Yellow Light as a whole group discuss the answer to question 7 before moving on to the Explain.

7. **Do you think that students should solve every problem involving rational functions symbolically? What understanding would students gain from solving with tables and graphs?**
   Responses may vary. Students need to have as many different possible ways to solve a problem as they can. Not only can they check their answer by solving differently, but they may be able to conceptually understand their solution when they can see it in a graph or in a table.

Explain

Leaders' Note: The Maximizing Algebra II Performance (MAP) professional development is intended to be an extension of the ideas introduced in Mathematics TEKS Connections (MTC). Throughout the professional development experience, we will allude to components of MTC such as the Processing Framework Model, the emphasis of making connections among representations, and the links between conceptual understanding and procedural fluency.

Debriefing the Experience:

1. **What concepts did we explore in the previous set of activities? How were they connected?**
   Responses may vary and may include generating rational functions to describe data and transforming rational functions.

2. **What procedures did we use to work with rational functions and equations? How are they related?**
   Tabular, graphical, and symbolic procedures (including procedures to solve rational equations) were all used throughout the Explore phase. Ultimately, they are all connected through the numerical relationships used to generate them.

3. **What knowledge from Algebra I do students bring about rational functions?**
   According to the Algebra I TEKS, students model inverse variation using rational functions.

4. **After working with rational functions in Algebra II, what are students' next steps in Precalculus or other higher mathematics courses?**
   According to the Precalculus TEKS, students will continue their study of continuity and end-behavior, asymptotes, and limits.

Anchoring the Experience:

5. **Distribute to each table group a poster-size copy of the Processing Framework Model.**

6. **Ask each group to respond to the question:**

   Where in the processing framework would you locate the different activities from the Explore phase?

7. **Participants can use one color of sticky notes to record their responses.**

Horizontal Connections within the TEKS

8. **Direct the participants' attention to the second layer in the Processing Framework Model: Horizontal Connections among Strands.**

9. **Prompt the participants to study the Algebra II TEKS and record those TEKS that connect to this Explore/Explain cycle. Prompt participants to attend to both the knowledge statements as well as the student expectations.**

10. Invite each table group to share 2 connections that they found and record them so that they are visible to the entire group.

Vertical Connections within the TEKS

11. Direct the participants' attention to the third layer in the Processing Framework Model: Vertical Connections across Grade Levels.

12. Prompt the participants to study the Algebra I, Geometry, Math Models, and Precalculus TEKS and record those TEKS that connect to this Explore/Explain cycle. Prompt participants to attend to both the knowledge statements as well as the student expectations.

13. Invite each table group to share 2 connections that they found, recording so that the entire large group may see.

14. Provide each group of participants with a clean sheet of chart paper. Ask them to create a "mind map" for the mathematical term "rational functions."

15. Provide an opportunity for each group to share their mind maps with the larger group. Discuss similarities, differences, and key points brought forth by participants.

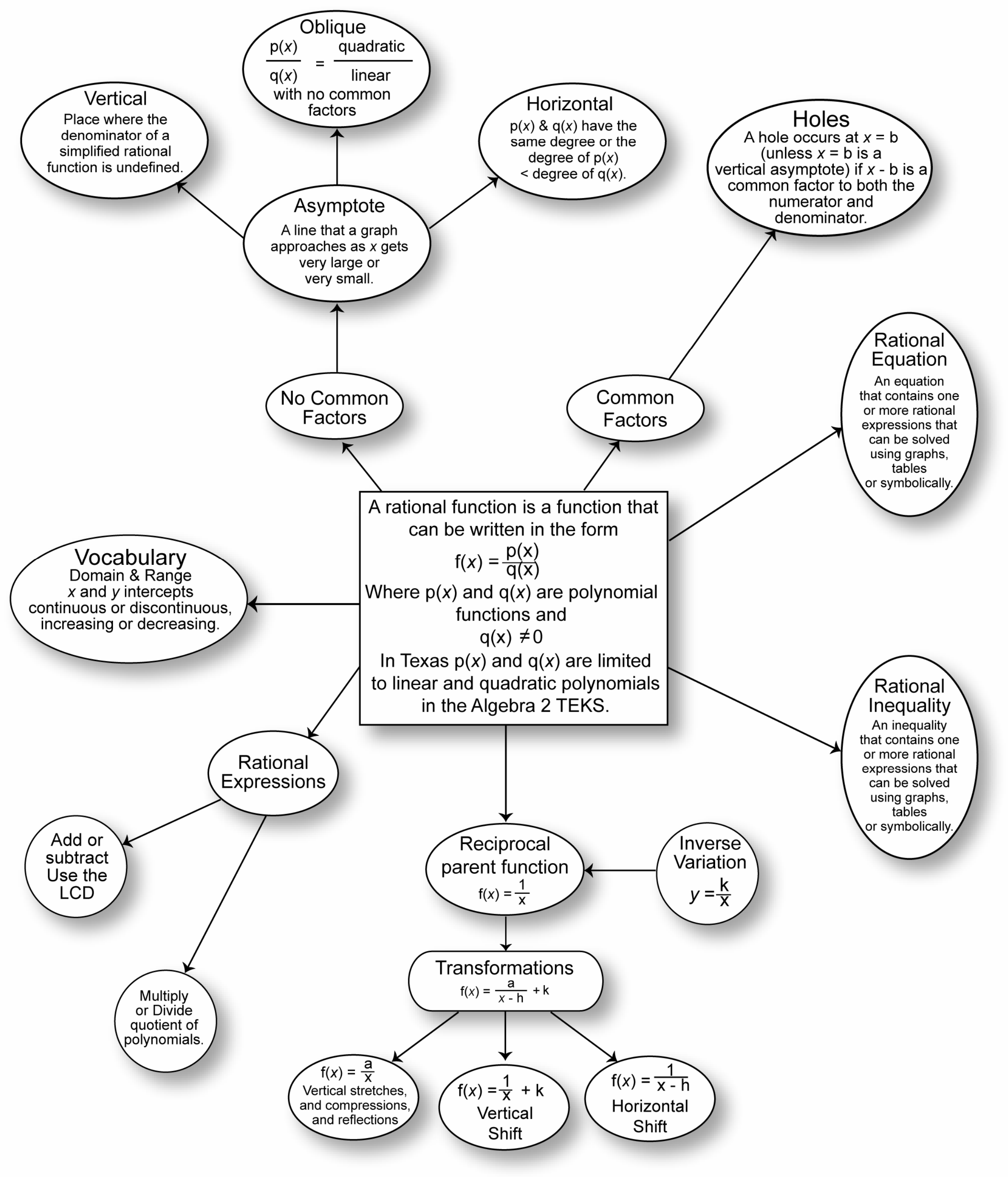
16. Distribute the vocabulary organizer template to each participant. Ask participants to construct a vocabulary model for the term rational functions.

17. When participants have completed their vocabulary models, ask participants to identify strategies from their experiences so far in the professional development that could be used to support students who typically struggle with Algebra II topics.

Note to Leader: *You may wish to have each small group brainstorm a few ideas first, then share their ideas with the large group while you record their responses on a transparency or chart paper.*

18. How would this lesson maximize student performance in Algebra II for teaching and learning the mathematical concepts and procedures associated with rational functions?
Responses may vary. Anchoring procedures within a conceptual framework helps students understand what they are doing so that they become more fluent with the procedures required to accomplish their tasks. Problems present themselves in a variety of representations; providing students with multiple procedures to solve a given problem empowers students to solve the problem more easily.

Rational Functions Mind Map



Vocabulary Organizer

Rational Functions

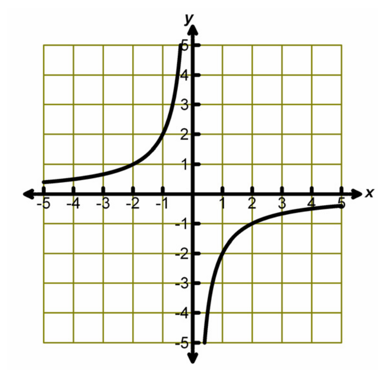
My definition

can be written in the form $\frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomials and q(x) ≠0

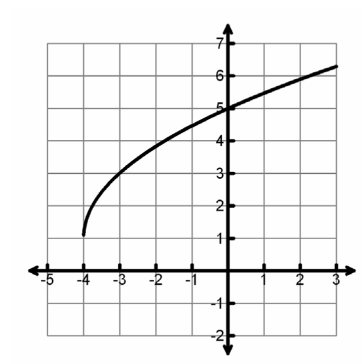
Personal Association

Similar to operations with fractions, may need a common denominator, may need to multiply by the reciprocal, the denominator cannot equal zero

Example



Non-Example



Elaborate

Leaders' Note: *In this phase, participants will extend their learning experiences to their classroom.*

1. **Distribute the 5E Student Lesson planning template. Ask participants to think back to their experiences in the Explore phase. Pose the following task:**

   What might a student-ready 5E lesson on inverses of relations/functions look like?
   - ❑ **What would the Engage look like?**
   - ❑ **Which experiences/activities would students explore firsthand?**
   - ❑ **How would students formalize and generalize their learning?**
   - ❑ **What would the Elaborate look like?**
   - ❑ **How would we evaluate student understanding of inverses of relations/functions?**

2. **After participants have recorded their thoughts, direct them to the student lesson for inverses of relations/functions. Allow time for participants to review lessons.**

3. **How does this 5E lesson compare to your vision of a student-centered 5E lesson?**
   Responses may vary.

4. **How does this lesson help remove obstacles that typically keep students from being successful in Algebra II?**
   By connecting symbolic manipulation to conceptual understanding as revealed in other representations (such as graphing), students have other tools with which to solve meaningful problems.

5. **How does this lesson maximize your instructional time and effort in teaching Algebra II?**
   Taking time to create a solid conceptual foundation reduces the need for re-teaching time and effort.

6. **How does this lesson maximize student learning in Algebra II?**
   Using multiple representations and foundations for functions concepts allows students to make connections among different ideas. These connections allow students to apply their learning to new situations more quickly and readily.

7. **How does this lesson accelerate student learning and increase the efficiency of learning?**
   Foundations for functions concepts such as function transformations transcend all kinds of functions. A basic toolkit for students to use when working with functions allows students to rethink what they know about linear and quadratic functions while they are learning concepts and procedures associated with other function families.

8. **Read through the suggested strategies on Strategies that Support English Language Learners. Consider the possible strategies designed to increase the achievement of English language learners.**

As participants read through the strategies that support English language learners and strategies that support students with special needs, they may notice that eight of the ten strategies are the same. The intention is to underscore effective teaching practices for all students. However, English language learners have needs specific to language that students with special needs may or may not have. The two strategies that are unique to the English language learners reflect an emphasis on language. Students with special needs may have prescribed modifications and accommodations that address materials and feedback. Students with special needs often benefit from progress monitoring with direct feedback and adaptation of materials for structure and/or pacing. A system of quick response is an intentional plan to gather data about a student's progress to determine whether or not the modification and (or) accommodation are (is) having the desired effect. The intention of the strategies is to provide access to rigorous mathematics and support students as they learn rigorous mathematics.

9. **What evidence of these strategies do you find in this portion of the professional development?**
   Responses may vary. ***Note:*** *Some strategies reflect teacher behaviors. The presenter may need to prompt participants to consider how the professional development materials support the needed teacher behaviors. For example, a student lesson may outline a structured approach for exploration so that the activity is non-threatening. This contributes to the teacher's ability to create an emotionally safe environment for learning.*

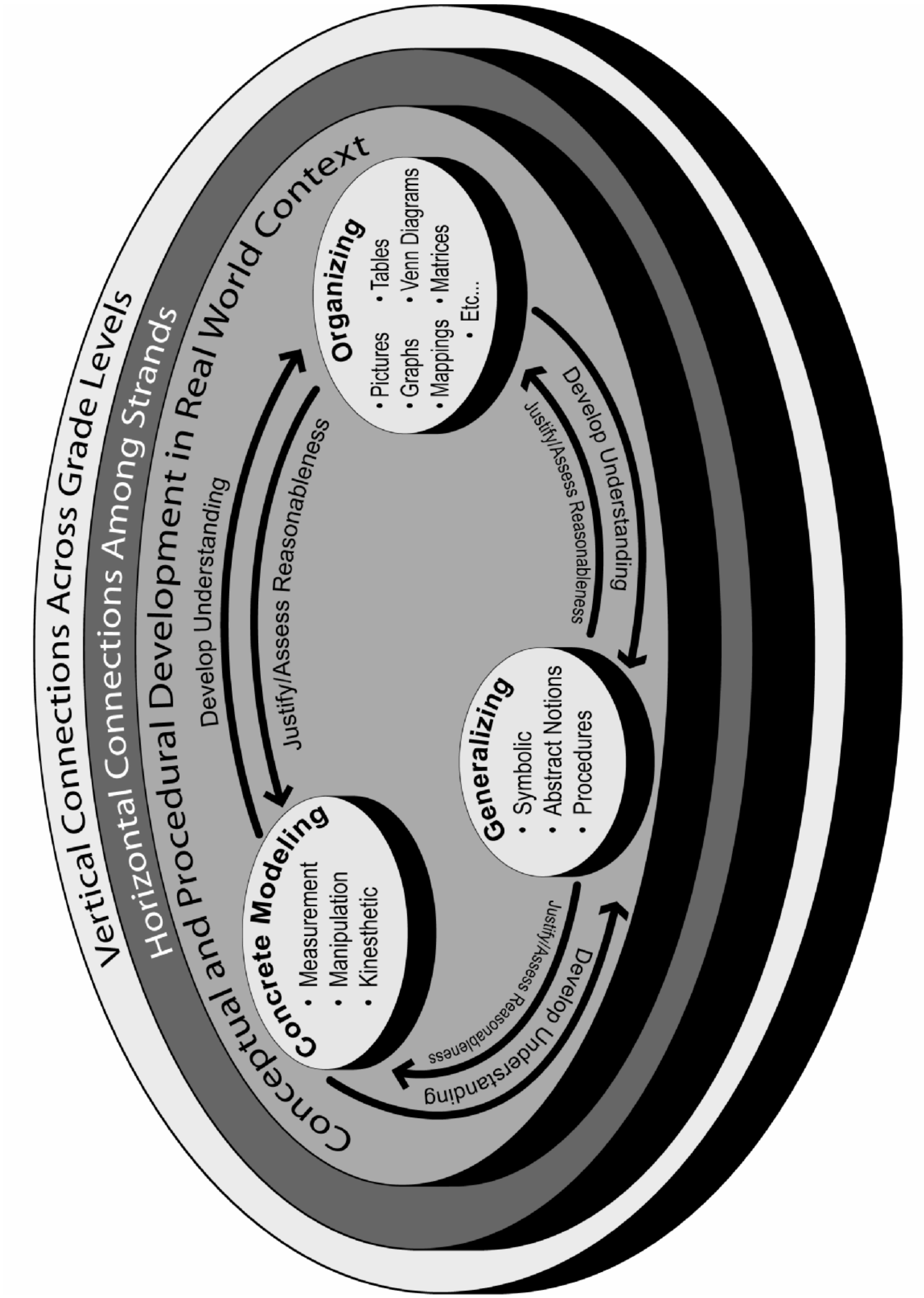
10. **Which strategies require adaptation of the materials in this portion of the professional development?**
    Responses may vary.

11. **Read through the suggested strategies on Strategies that Support Students with Special Needs. Consider the possible strategies designed to increase the achievement of students with special needs.**

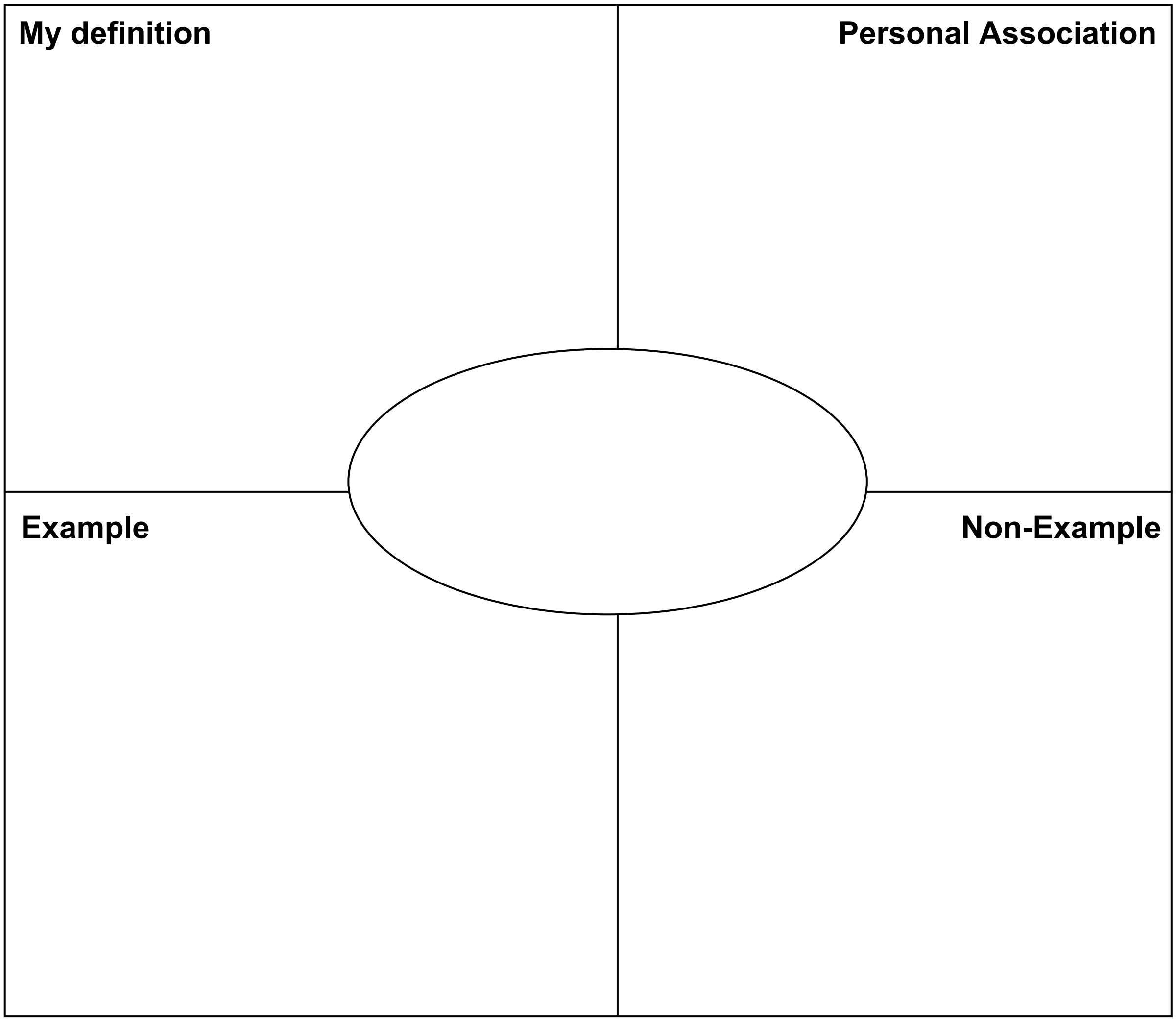
12. **What evidence of these strategies do you find in this portion of the professional development?**
    Responses may vary. ***Note:*** *Some strategies reflect teacher behaviors. The presenter may need to prompt participants to consider how the professional development materials support the needed teacher behaviors. For example, a student lesson may outline a structured approach for exploration so that the activity is non-threatening. This contributes to the teacher's ability to create an emotionally safe environment for learning.*

13. **Which strategies require adaptation of the materials in this portion of the professional development?**
    Responses may vary.

Processing Framework Model



Vocabulary Organizer



5E Student Lesson Planning Template

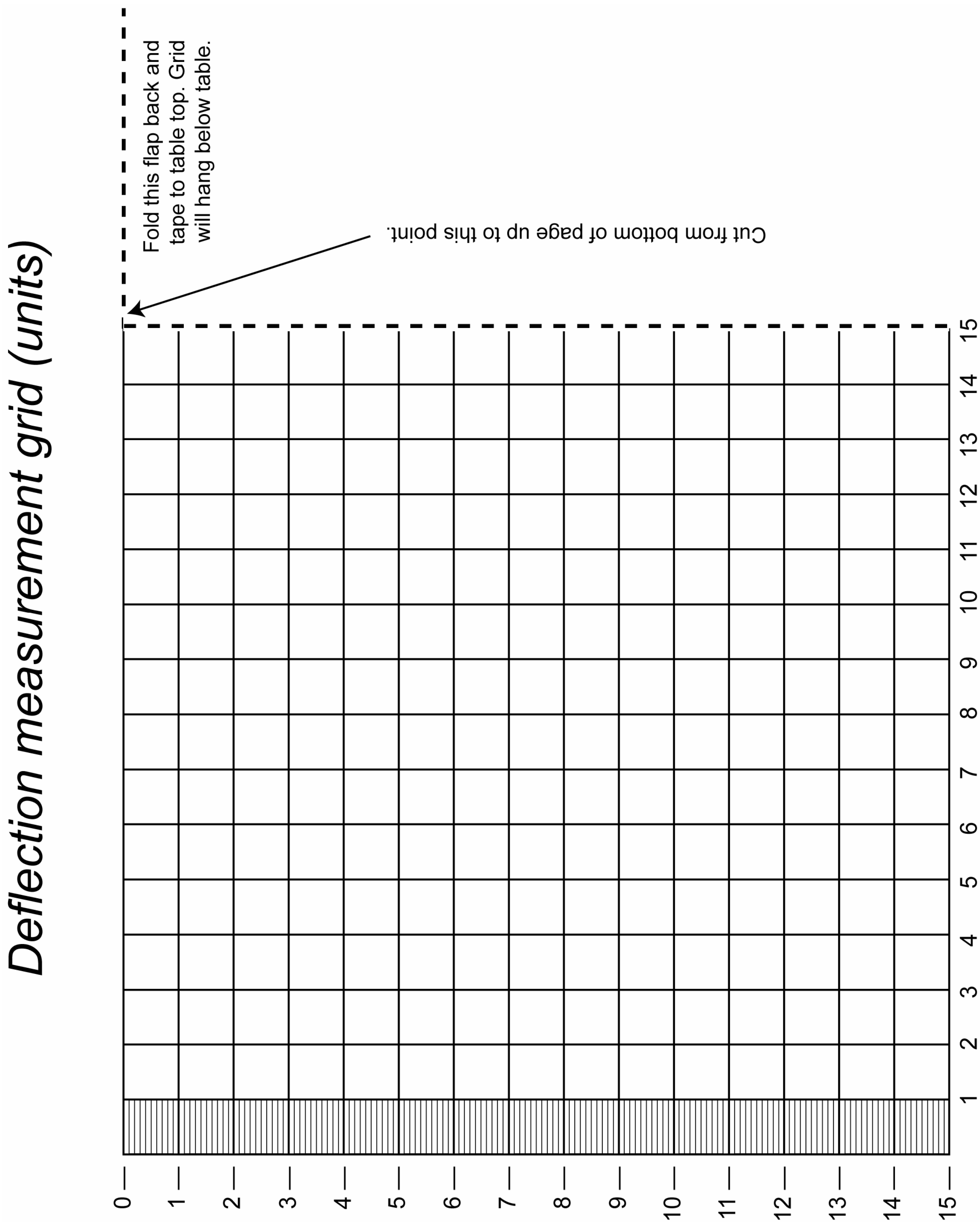
Description	Activity
Engage The activity should be designed to generate student interest in a problem situation and to make connections to prior knowledge. The instructor initiates this stage by asking meaningful questions, posing a problem to be solved, or by showing something intriguing.	
Explore The activity should provide students with an opportunity to become actively involved with the key concepts of the lesson through a guided exploration requiring them to probe, inquire, and question. The instructor actively monitors students as they interact with each other and the activity.	
Explain Students collaboratively begin to sequence events/facts from the investigation and communicate these findings to each other and the instructor. The instructor, acting in a facilitation role, formalizes student findings by providing further explanations and additional meaning or information, such as correct terminology.	
Elaborate Students extend, expand, or apply what they have learned in the first three stages and connect this knowledge with prior learning to deepen understanding. Instructors can use the Elaborate stage to verify students' understandings.	
Evaluate Evaluation occurs throughout students' learning experiences. More formal evaluation can be conducted at this stage. Instructors can determine whether the learner has reached the desired level of understanding the key ideas and concepts.	

Strategies that Support English Language Learners (ELL)

Strategy	Explore, Explain, Elaborate 4
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond.	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Be conscious of tone and diction. Speak slowly and distinctly.	
Incorporate language skills (reading, writing, speaking, and listening) into instruction.	

Strategies that Support Students with Special Needs

Strategy	Explore, Explain, Elaborate 4
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond.	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Use a system of quick response to needs and accommodations including progress monitoring to inform instruction.	
Accommodate materials for format, structure, sequence, etc. as needed.	


Deflection measurement grid (units)
Fold this flap back and tape to table top. Grid will hang below table.
Cut from bottom of page up to this point.
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Transparency

Average Amount of Deflection in the Linguine Bundles

Number of Pieces of Linguine in the Bundle	Amount of Deflection for Each Team												Average
	A	B	C	D	E	F	G	H	I	J	K	L	
1													
2													
3													
4													
5													
6													
7													
8													

Activity Page

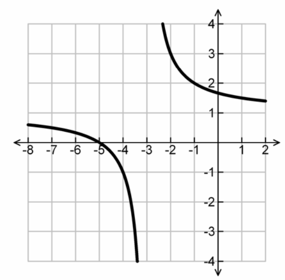
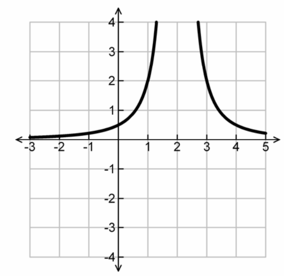
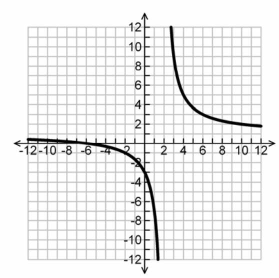
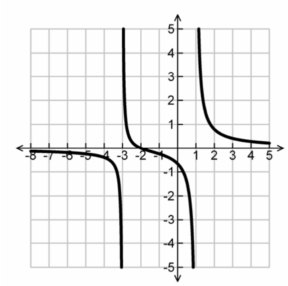
Rational Function Card Sort

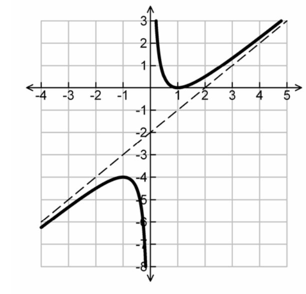
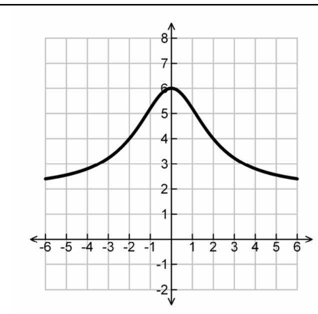
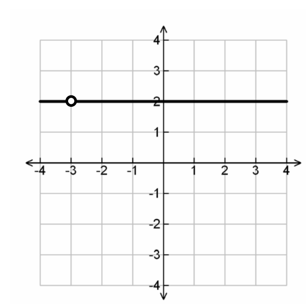
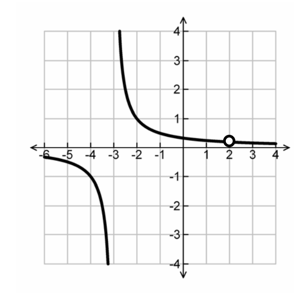
$b(x) = x$	$c(x) = e^x$
$d(x) = 6 + \frac{10}{x-2}$	$f(x) = 7 - 3\sqrt{x-2}$
$g(x) = 3^{x-4} + 10$	$h(x) = \frac{x^2 + 6x - 3}{x+4}$
$j(x) = -\frac{1}{4}$	$k(x) = \sqrt{2}x$
$m(x) = \log(2x+2) - 10$	$n(x) = \frac{2x+3}{3x-1}$

$p(x)=\sqrt{3}x^2+2x-7$	$q(x)=2x^2+3$
$r(x)=\sqrt{2x+1}$	$s(x)=10^{2x}+4$
$t(x)=\frac{2}{3}x^2-\frac{3}{4}$	$u(x)=\frac{\sqrt{x+2}}{\sqrt{2x-1}}$
$v(x)=\ln\left(\ln 2x+\frac{2}{3}\right)-\frac{3}{4}$	$w(x)=\frac{\sqrt{2}x^2+4}{\sqrt{10}x^2-4x+\sqrt{15}}$
$y(x)=5\frac{1}{2}$	$z(x)=(3+i)x^2+2ix$

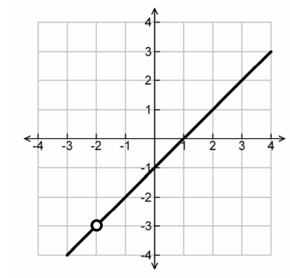
Rational Function Card Match

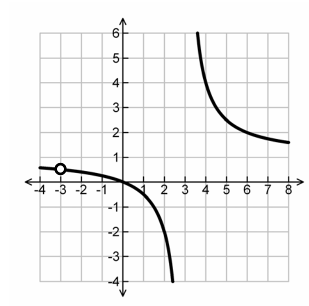
Activity Cards

$f(x)=\dfrac{x+5}{x+3}$ **A**	$f(x)=\dfrac{2}{x+3}+1$	$x=-3$ $y=1$ x-intercept at $(-5, 0)$ y-intercept at $(0, \frac{5}{3})$	
$f(x)=\dfrac{2}{x^2-4x+4}$ **B**	$f(x)=\dfrac{2}{(x-2)^2}$	$x=2$ $y=0$ no x-intercepts y-intercept at $(0, 0.5)$	
$f(x)=\dfrac{x+6}{x-2}$ **C**	$f(x)=\dfrac{8}{x-2}+1$	$x=2$ $y=1$ x-intercept at $(-6, 0)$ y-intercept at $(0, -3)$	
$f(x)=\dfrac{x+2}{x^2+2x-3}$ **D**	$f(x)=\dfrac{x+2}{(x+3)(x-1)}$	$x=1$ and $x=-3$ $y = 0$ x-intercept at $(-2, 0)$ y-intercept at $(0, -\frac{2}{3})$	

$f(x)=\dfrac{x^2-2x+1}{x}$ **E**	$f(x)=x-2+\dfrac{1}{x}$	$y=x-2$ $x=0$ x-intercept at (1, 0) no y-intercepts	
$f(x)=\dfrac{2x^2+24}{x^2+4}$ **F**	$f(x)=\dfrac{16}{x^2+4}+2$	$y=2$ no vertical asymptotes no x-intercepts y-intercept at (0, 6)	
$f(x)=\dfrac{2x+6}{x+3}$ **G**	$f(x)=2,$ $x\neq -3$	no horizontal asymptote removable discontinuity (hole) at (−3, 2) no x-intercepts y-intercept at (0, 2)	
$f(x)=\dfrac{x-2}{x^2+x-6}$ **H**	$f(x)=\dfrac{1}{x+3},$ $x\neq -2$	x = –3 and y = 0 removable discontinuity (hole) at (2, 0.2) no x-intercepts y-intercept at (0, $\frac{1}{3}$)	

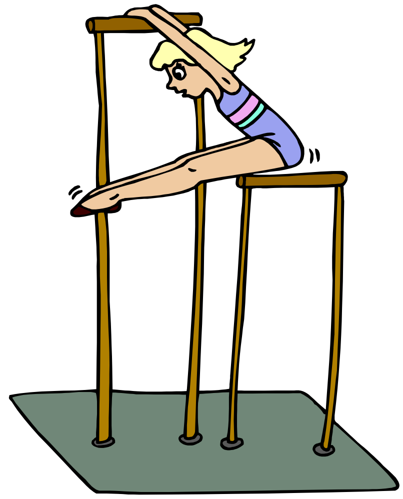
<table>
<tr><td rowspan="4">$f(x)=\dfrac{x^2+x-2}{x+2}$

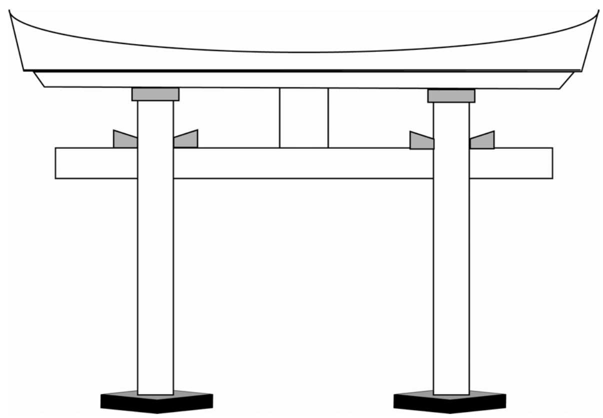
I</td><td rowspan="4">$f(x)=x-1,$
$x\neq -2$</td><td>no horizontal asymptote</td><td rowspan="4"></td></tr>
<tr><td>removable discontinuity (hole) at (−2, −3)</td></tr>
<tr><td>x-intercept at (1, 0)</td></tr>
<tr><td>y-intercept at (0, −1)</td></tr>
<tr><td rowspan="4">$f(x)=\dfrac{x^2+3x}{x^2-9}$

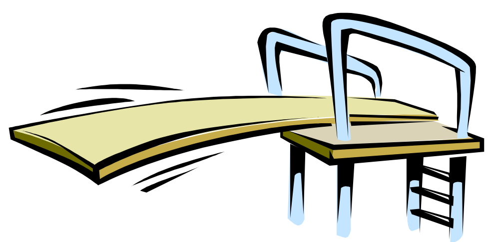
J</td><td rowspan="4">$f(x)=\dfrac{3}{x-3}+1,$
$x\neq -3$</td><td>x = 3 and $y=1$</td><td rowspan="4"></td></tr>
<tr><td>removable discontinuity (hole) at (−3, 0.5)</td></tr>
<tr><td>x-intercept at (0, 0)</td></tr>
<tr><td>y-intercept at (0, 0)</td></tr>
</table>

Participant Pages: Rational Functions

How are the objects below alike? How are they different?







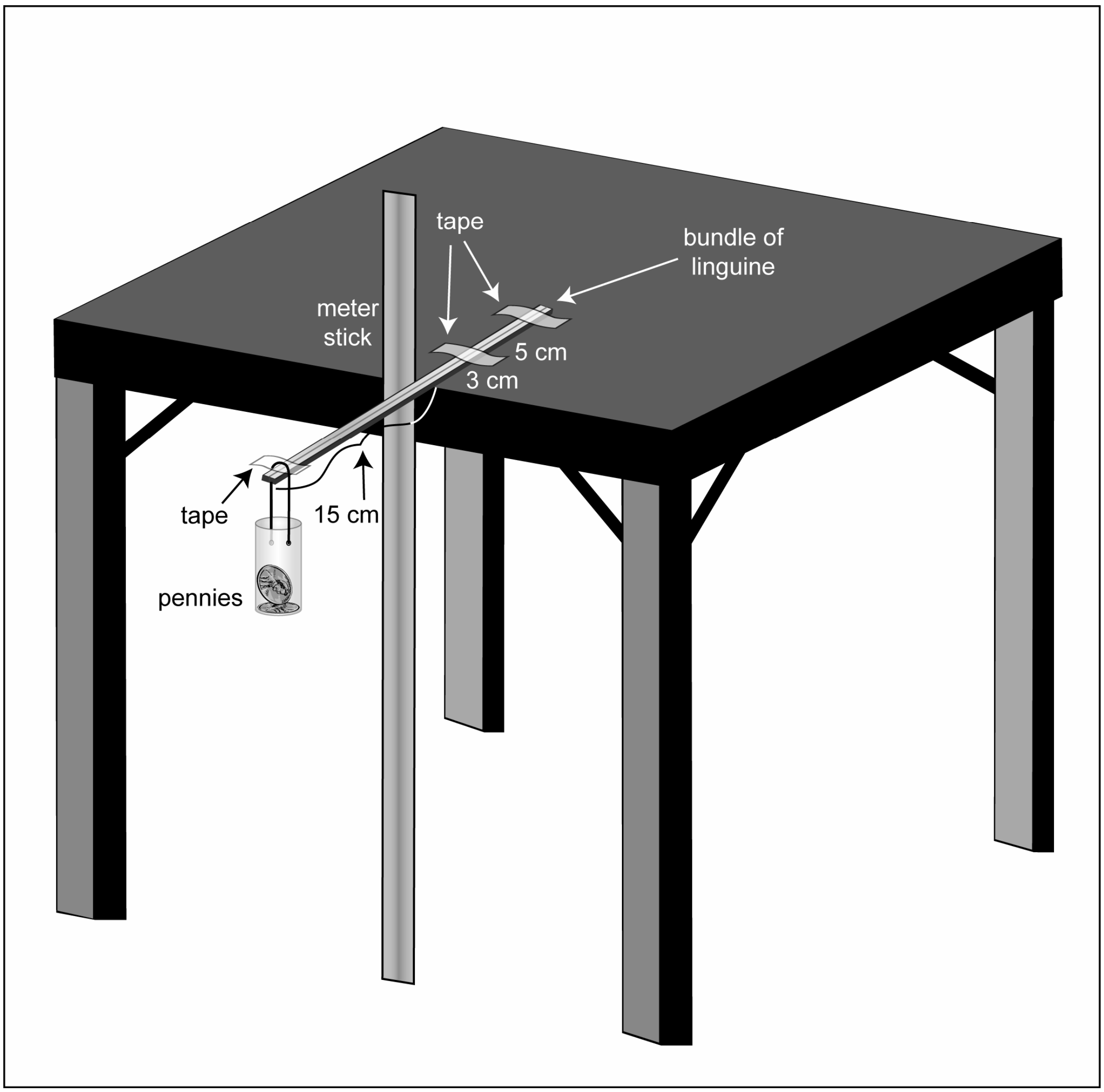
<u>Part 1</u>: Linguine Cantilever

A cantilever is a projecting structure that is secured at only one end and carries a load on the other end. Diving boards and airplane wings are examples of horizontal cantilevers. Flagpoles and chimneys are vertical cantilevers. One of the most famous examples of a cantilever in architecture, which is shown below, is the Frank Lloyd Wright designed home, Fallingwater. The strength of a cantilever can be affected by variables such as length, load, cross sectional area, temperature, or elasticity. In this activity, you will be investigating the relationship between the thickness of a cantilever and the deflection in the cantilever when weight is added at the end.

In this investigation, you will keep the length of a piece of linguine that is hanging over the edge of a desk constant as you collect data on how much the linguine deflects. Deflection is the amount that the linguine bends in the downward direction. The number of pieces of linguine will change. Since you want to keep all variables (except for the ones you are investigating) constant make sure to pay attention to the hints listed with the instructions.

Have participants get into groups of three. Each person in the group has a job.

Materials manager:	Get the necessary materials, direct the team in setting up the investigation
Measures manager:	Measure the amount of deflection as the investigation proceeds
Data manager:	Record the necessary measurements in the table, share the data with the team



Data Collection Set-up Instructions

Step 1. The materials person should get the necessary materials and begin to make bundles of linguine. Each bundle of 1, 2, 3, 4, 5, 6, 7, and 8 pieces of linguine should be taped one inch from each end. Since linguine is not all exactly the same length, try to keep one end of the bundle lined up.

Step 2. Tape a short piece of string to the 35mm film canister to form a handle. If you do not have a film canister, use a baggie, transparent tape, and a paper clip to build a weight to hang from the linguine.

Step 3. Tape one piece of linguine with 15 centimeters hanging over the edge of a desk. Put one piece of tape approximately 3 cm from the edge of the table. Place a second piece of tape over the end of linguine. Place the load (film canister) on the end of the linguine that is hanging over the edge of the desk. Slowly place pennies in the film canister until the linguine breaks. Wait 15 seconds before adding an additional penny. Use one less penny than the number required to break one piece of linguine as the load in your bucket for the remainder of this data collection experiment.

Step 4. Tape a meter stick perpendicularly to the floor next to a desk.

Step 5. Measure the linguine's height above the floor without the film canister attached. (Hint: It is easier to consistently measure the height using the bottom of the linguine.)

Step 6. Place your pennies into the bucket. (Hint: Place the pennies gently, throwing pennies into the bucket will alter the results.)

Step 7. Place the bucket on the end of the linguine that is hanging over the edge of the desk. (Hint: Place the string at the same point on the linguine for each trial. Use a piece of masking tape to hold the bucket onto the linguine.)

Step 8. Wait 15 seconds. Measure the amount of deflection in the linguine. (Hint: The easiest method for measuring deflection is to use the eraser end of a pencil to line up the deflection of the end of the linguine with its measure on the meter stick.) Record your measurements in the table.

Step 9. Repeat the procedure with two pieces of linguine taped together still hanging 15 centimeters over the edge of the desk. Measure the deflection of the bundle of linguine.

Step 10. Continue repeating the procedure with additional pieces of linguine until you measure deflection with eight pieces taped together. Continue to record your data.

1. Fill in the table with the data you collected.

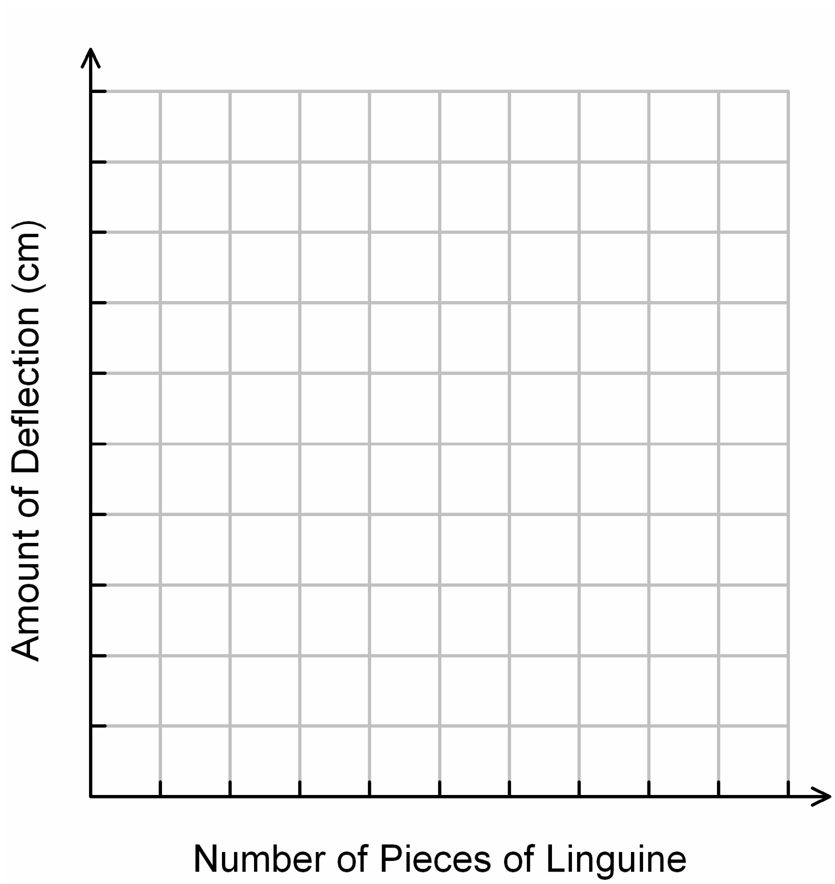
Number of Pieces of Linguine in the Bundle (x)	Starting Height of Linguine Bundle Above the Floor	Height of Linguine Bundle Above the Floor After the Load is Placed	Amount of Deflection in the Linguine (y)	Product of x and y ($x \cdot y$)
1				
2				
3				
4				
5				
6				
7				
8				

2. Write a dependency statement relating the two variables.

3. What is a reasonable domain for the set of data?

4. What is a reasonable range for the set of data?

5. Make a scatterplot of the data you collected.



6. Verbally describe what happens in this data collection investigation.

7. Is this data set continuous or discrete? Why?

8. Does the set of data represent a function? Why?

9. Does the data appear to be a linear, quadratic, exponential or some other type of parent function? Why do you think so?

10. Is the function increasing or decreasing?

11. Is the rate of change constant for this set of data?

12. Determine a function rule that models the set of data you collected.

13. To get a better model, add your set of data to the data of the entire group. Each group should send their data manager to the overhead to fill in the data collected for their group. Record the additional data in the table below. Find the average deflection for each bundle of linguine for the entire group.

Number of Pieces of Linguine in the Bundle	**Amount of Deflection for Each Team**												**Average**
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	
1													
2													
3													
4													
5													
6													
7													
8													

14. Using the entire group's data, what function would you now use to model this situation?

15. How does this investigation connect to the TEKS from previous courses?

16. What are the key points students need to understand about the Linguine Cantilever before continuing the investigation of rational functions?

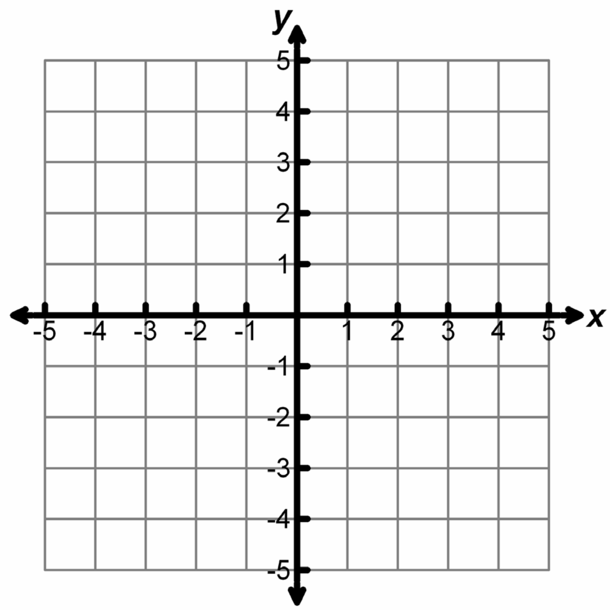
<u>Part 2</u>. Transformations to $f(x)=\dfrac{1}{x}$

1. What is the reciprocal of the linear parent function, $f(x)=x$?

2. Let's investigate some of the attributes of the function and its reciprocal. Fill in the tables with several values for each function. Draw a sketch of the graphs of the two functions on the same set of axes.

$f(x)=x$	
x	***y***
–3	
–2	
–1	
–0.5	
–0.1	
0	
0.1	
0.5	
1	
2	
3	

$f(x)=\dfrac{1}{x}$	
x	***y***
–3	
–2	
–1	
–0.5	
–0.1	
0	
0.1	
0.5	
1	
2	
3	



3. Using your graphing calculator (if necessary), fill in the tables below. Let $f(x)=x$ be Y_1, and let $g(x)=\dfrac{1}{x}$ be Y_2.

$$Y_1 = x$$

$$Y_2 = \frac{1}{x}$$

$Y_1 = x$		$Y_2=\dfrac{1}{x}$
	Intervals where the function is increasing	
	Intervals where the function is decreasing	
	Intervals where the function is undefined	
	Coordinates of the *x*-intercepts (zeros)	
	Equations of any asymptotes	

4. What do you notice about the graphs of the linear parent function and its reciprocal?

5. Where do the linear parent function and its reciprocal intersect?

6. How could you have your students investigate what happens to $f(x)$ as x gets closer and closer to 0 using the graphing calculator?

7. How could you have your students investigate what happens to $f(x)$ as x gets larger and larger?

8. How do the Algebra II TEKS name this new parent function?

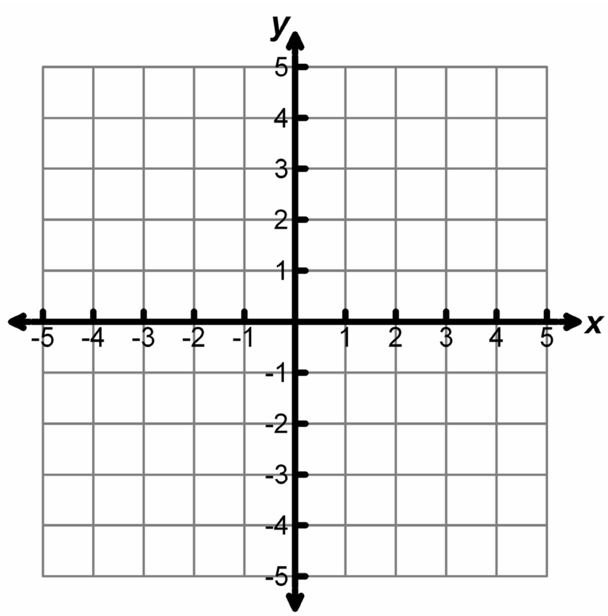
9. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x) = \frac{1}{x}$, when it is multiplied by a constant as in the examples below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results.

$$Y_1 = \frac{1}{x}$$

$$Y_2 = \frac{3}{x}$$

$$Y_3 = \frac{0.1}{x}$$

x	$Y_1 = \frac{1}{x}$	$Y_2 = \frac{3}{x}$	$Y_3 = \frac{0.1}{x}$
−3			
−2			
−1			
0			
1			
2			
3			
4			



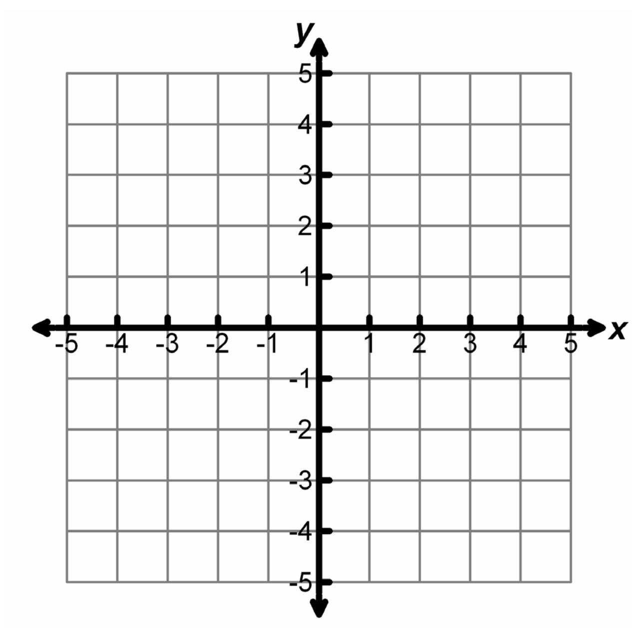
10. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x)=\frac{1}{x}$, when it is multiplied by a negative constant as in the examples below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results which includes any asymptotes.

$Y_1=\frac{1}{x}$

$Y_2=-\frac{1}{x}$

$Y_3=-\frac{4}{x}$

x	$Y_1=\frac{1}{x}$	$Y_2=-\frac{1}{x}$	$Y_3=-\frac{4}{x}$
-3			
-2			
-1			
0			
1			
2			
3			
4			



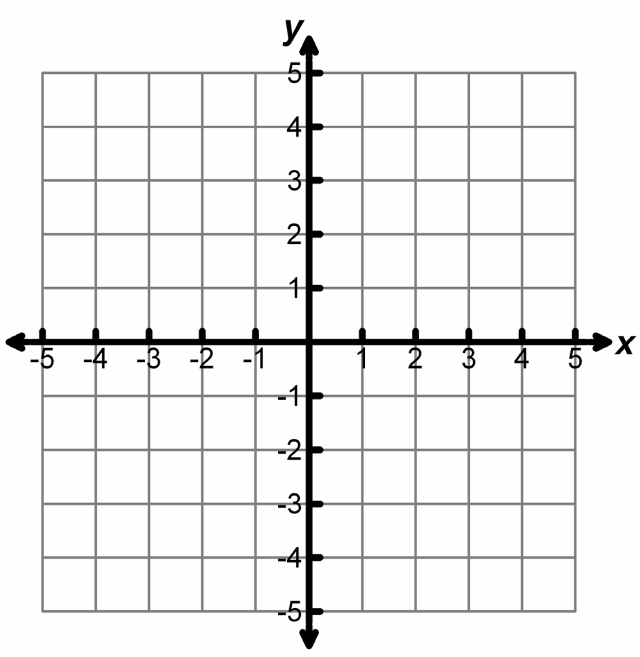
11. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x)=\frac{1}{x}$, if a constant is added to the function, as in the three functions listed below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results which includes any asymptotes.

$$Y_1=\frac{1}{x}$$

$$Y_2=\frac{1}{x}+3$$

$$Y_3=\frac{1}{x}-2$$

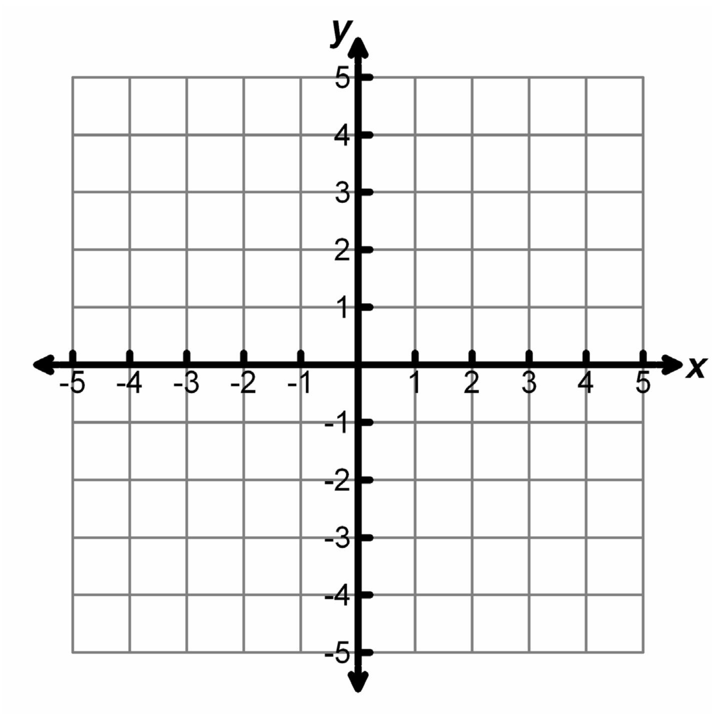
x	$Y_1=\frac{1}{x}$	$Y_2=\frac{1}{x}+3$	$Y_3=\frac{1}{x}-2$
–3			
–2			
−1			
0			
1			
2			
3			
4			



12. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x)=\frac{1}{x}$, if a constant is added to the x-coordinate in the denominator, as in the three functions listed below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results which includes any asymptotes.

$$Y_1=\frac{1}{x}$$
$$Y_2=\frac{1}{x+3}$$
$$Y_3=\frac{1}{x-2}$$

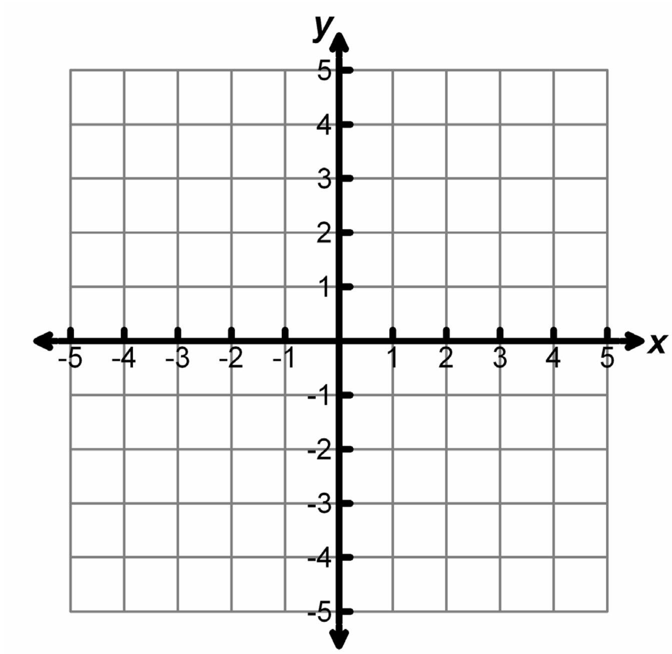
x	$Y_1=\frac{1}{x}$	$Y_2=\frac{1}{x+3}$	$Y_3=\frac{1}{x-2}$
-3			
-2			
-1			
0			
1			
2			
3			
4			



13. Predict, describe and then sketch the transformations to the reciprocal parent function in the function below.

$$f(x) = -\frac{2}{x+1} - 3$$

x	$Y_1 = \frac{1}{x}$	$Y_1 = -\frac{2}{x}$	$Y_1 = -\frac{2}{x+1}$	$Y_2 = -\frac{2}{x+1} - 3$
−3				
−2				
−1				
0				
1				
2				
3				
4				



14. Describe how transformations to the reciprocal function are similar to transformations to other parent functions.

15. What are the key points students need to understand about transformations to the reciprocal function before continuing the investigation of rational functions?

<u>Part 3</u>: Card Sort and Match

1. Sort the first deck of cards into two (and only two) groups. Describe your method for sorting.

2. Using the cards match the rational form of each function to the transformation form or factored form, the equations of any asymptotes, the coordinates of any removable discontinuities (holes), and the graph of the function. Record your answers in the table.

Card Match

Rational Function Form	Transformation Form	Discontinuities, Asymptotes, Other Noteworthy Points	Graph
A			
B			
C			

D			
E			
F			
G			

H			
I			
J			

3. What are the key points students need to understand about rational functions to be able to do the Rational Function Card Match?

<u>Part 4</u>: Length of a Yellow Light

One of the formulas traffic engineers use to help them calculate the length of time a traffic light should remain yellow is

$$Y(t) = t + \frac{v}{2a} + \frac{w+L}{v}.$$

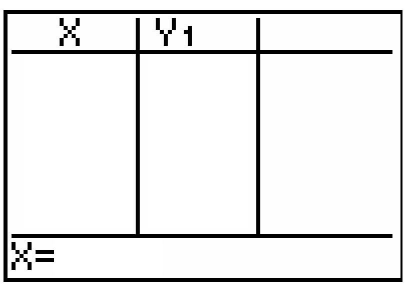
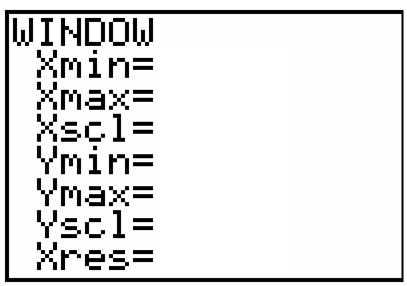
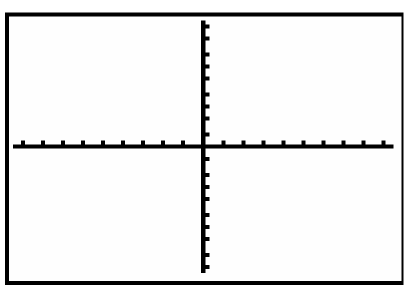
The formula takes into account reaction time, braking time, and intersection clearance time. The variables used in this calculation are:

- t = reaction time (usually 1 second)
- v = velocity of the vehicle (in feet/second)
- a = deceleration rate (approximately 10 feet/ sec^2)
- w = width of the intersection (feet)
- L = length of the vehicle (feet)

In order to calculate the speed limit for a certain intersection that is 48 feet wide, the engineer uses an average car length of 18 feet. She can calculate the length of time the traffic signal should remain yellow at that intersection based on the velocity in feet/second using the following formula:

$$Y(t) = 1 + \frac{v}{20} + \frac{66}{v}.$$

1. If the posted speed limit at the intersection is 55 miles per hour, how long should the signal remain yellow?

Solution using a table:	Solution using a graph:	Symbolic solution:
	 	

2. For a traffic signal to remain yellow for 4 seconds what should the department of transportation post as the speed limit?

3. If the speed of vehicles at a particular intersection varies between 30 and 50 mph, how long do you think the traffic signal should remain yellow?

4. A tractor trailer that is approximately 36 feet long travels through the same intersection when the signal remains yellow for 6 seconds. Based on the formula, how fast should the tractor trailer be allowed to drive through the intersection? How fast should a car be allowed to drive through the intersection?

$$Y(v) = 1 + \frac{v}{20} + \frac{84}{v}$$

5. Do you think this is a good model for length of time that traffic signals should remain yellow for every x value in the domain?

Bonus question:

6. What is the oblique asymptote for this rational function? Graph both the function and the asymptote on your graphing calculator.

$$Y(v) = 1 + \frac{v}{20} + \frac{66}{v}$$

7. Do you think that students should solve every problem involving rational functions symbolically? What understanding would students gain from solving with tables and graphs?