Setting the Stage

Ms. G has 8 years of experience teaching Algebra II. This year she has five sections of Algebra II filled with 11th and 12th grade students. Ten percent of her students are enrolled in Algebra II for the second time. Ten percent of her students speak little or no English and have Spanish or Vietnamese as their primary language. Twenty-five percent of her students are fluent in English and Spanish. Five percent of her students are fluent in English and Vietnamese. Ten percent of her students have IEPs requiring accommodations and/or modifications. Thirty percent of her students did not pass the TAKS as tenth graders. Forty percent of her students are classified as low SES. Ms. G is sometimes overwhelmed with finding approaches that address the needs of all of the students in her classes.

Ms. G prefers to plan whole group instruction. She is trying to incorporate small group instruction a few times each six weeks. Ms. G feels that she gets the best feedback from students when everyone is focused on one task together. She has recently started to have her students write about the mathematics they are studying.

Prior to this lesson, Ms. G reviewed linear, quadratic, and exponential parent functions to help students remember what they learned in Algebra I. She provided a brief explanation of the absolute value parent function as a non-example of the other functions. The students used graphing technology to graph the parent functions, perform transformations on the parent functions, and analyze the rates of change associated with the parent functions.

Ms. G's Lesson Plan

TEKS:

2A.1A The student uses properties and attributes of functions and applies functions to problem situations. The student is expected to identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.

2A.1B The student uses properties and attributes of functions and applies functions to problem situations. The student is expected to collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

Engage:
The U.S. Geological Survey (USGS) is dedicated to the timely, relevant, and impartial study of the landscape, our natural resources, and the natural hazards that threaten us. To accomplish this, the USGS collects real-time data about the depth of water in streams, bayous, ponds, and lakes in the United States. Why might they do this?
http://www.usgs.gov/

Explore:
1. Assign each group one of the four sets of data. Provide each group with the appropriate graph and table of data.
2. Prompt students to answer the questions on the activity page for their assigned graph and table of data.
3. Prompt students to create a poster that summarizes their learning.

Explain:
1. Have students share their group summaries.
2. Debrief using these questions.
   a. How did the reasonable domains compare for these different situations?
   b. How did the reasonable ranges compare for these different situations?
   c. Are the data continuous or discrete? Why?
   d. Which, if any, parts of these graphs can be modeled by a parent function? Why?

Elaborate:
1. Assign each group a different set of data.
2. Direct each group to analyze this new set of data. How does it compare to your original set of data? How is it different?

Evaluate:
Prompt students to complete 2 of the 3 sentence starters.
1. Understanding domain and range helps me to…
2. A parent function tells…
3. The rate of change of data helps us identify a possible parent function because…

Ms. G's Lesson Plan: Student Activity Page

1. What is the title of your set of data?

2. What do you notice about the graph of the data?

3. What is a reasonable domain for this situation? Why?

4. What is a reasonable range for this situation? Why?

5. Are the data continuous or discrete? Why?

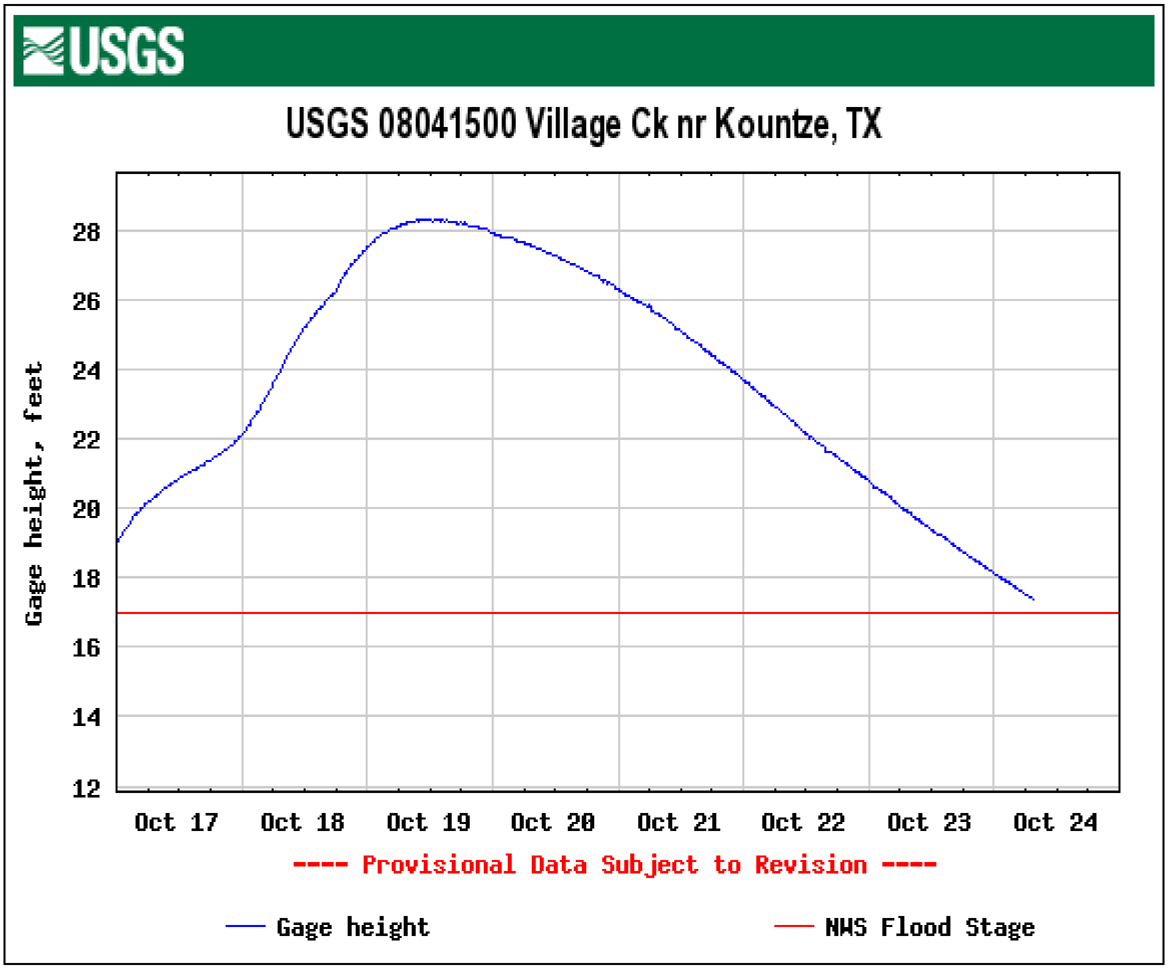
6. Which, if any, part of this graph can be modeled by a linear function? How do you know?

7. Which, if any, part of this graph can be modeled by a quadratic function? How do you know?

8. Which, if any, part of this graph can be modeled by an exponential function? How do you know?

9. Which, if any, part of this graph can be modeled by an absolute value function? How do you know?

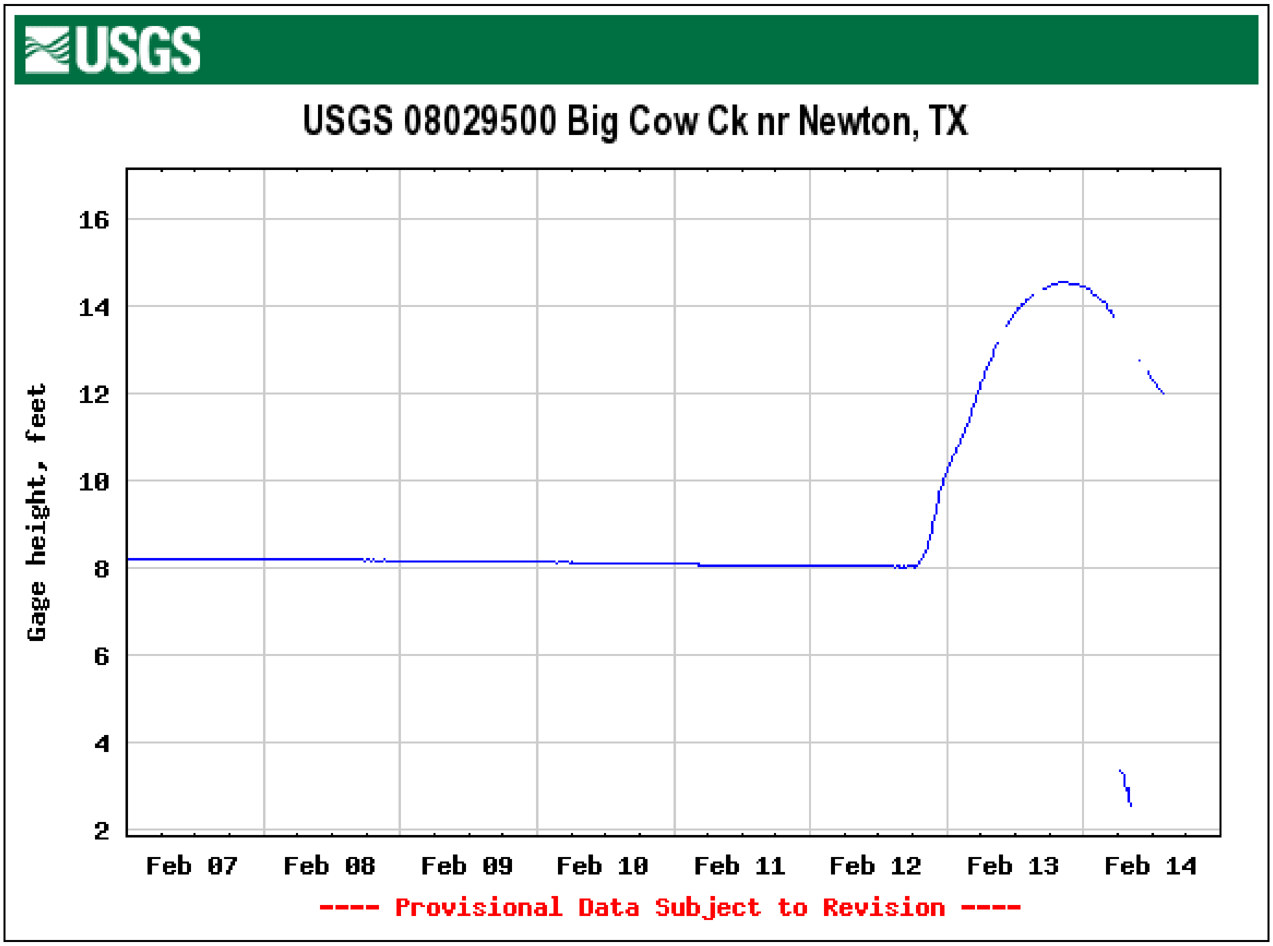
Ms. G's Lesson Plan: Data Set A



Date	Gage Height at Noon (ft)
Oct. 17	20.8
Oct. 18	25.4
Oct. 19	28.2
Oct. 20	27.1
Oct. 21	25.0
Oct. 22	22.1
Oct. 23	19.2

http://waterdata.usgs.gov/nwis/rt

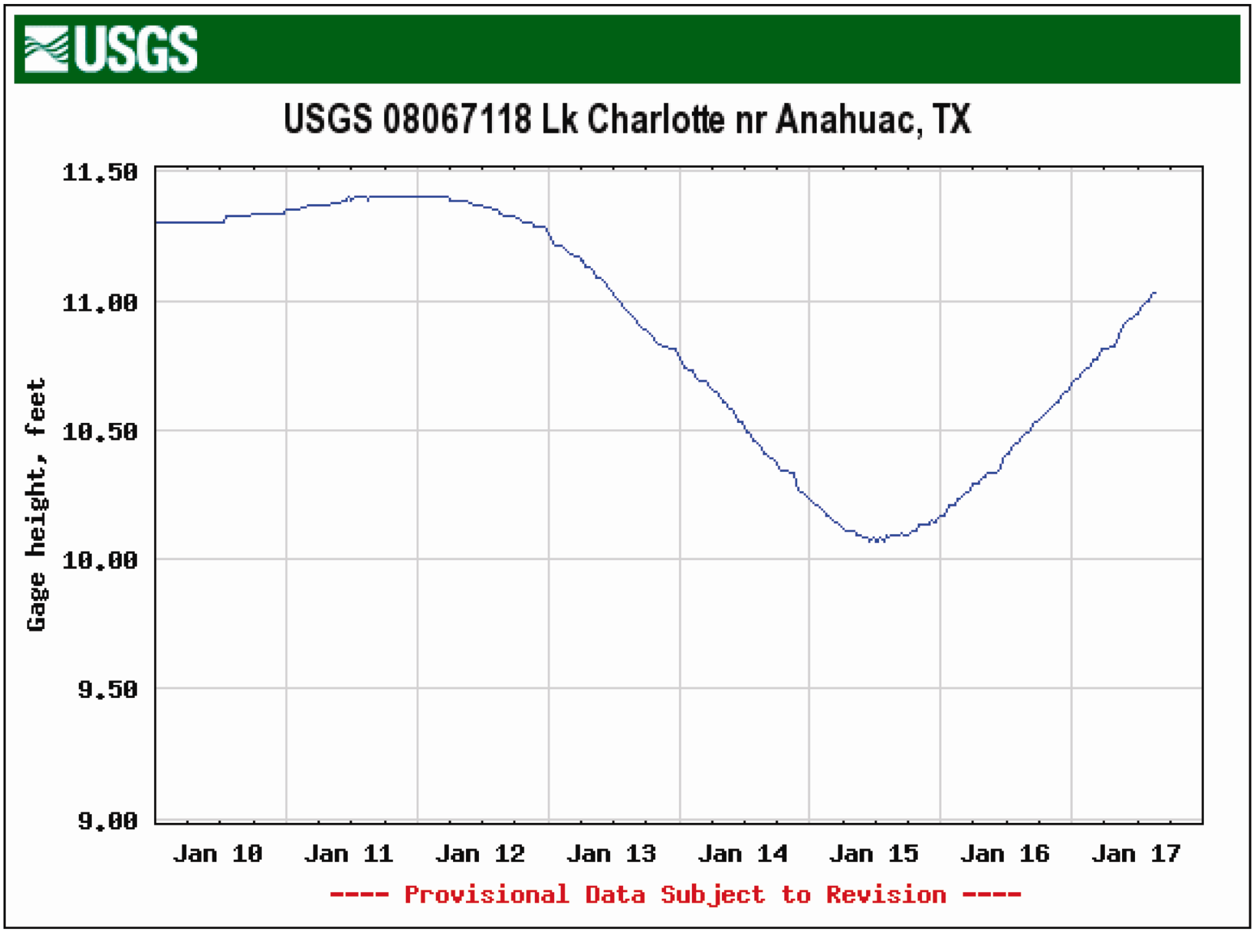
Ms. G's Lesson Plan: Data Set B



Date	Gage Height at Noon (ft)
Feb. 7	8.2
Feb. 8	8.2
Feb. 9	8.2
Feb. 10	8.1
Feb. 11	8.0
Feb. 12	8.0
Feb. 13	13.5
Feb. 14	12.1

http://waterdata.usgs.gov/nwis/rt

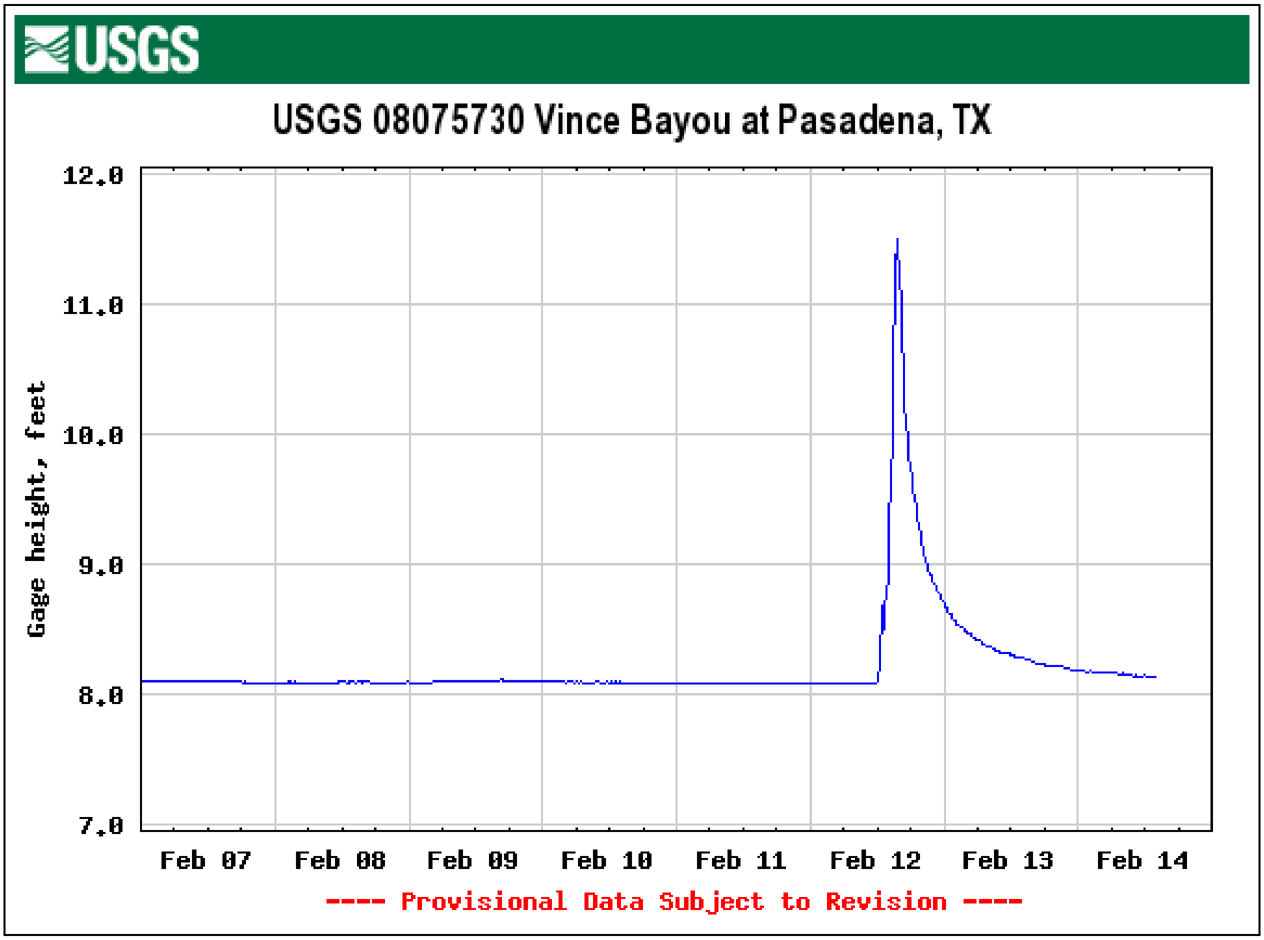
Ms. G's Lesson Plan: Data Set C



Date	Gage Height at Noon (ft)
Jan. 10	11.30
Jan. 11	11.35
Jan. 12	11.32
Jan. 13	11.08
Jan. 14	10.51
Jan. 15	10.10
Jan. 16	10.40
Jan. 17	10.90

http://waterdata.usgs.gov/nwis/rt

Ms. G's Lesson Plan: Data Set D



Date	Gage Height at Noon (ft)
Feb. 7	8.1
Feb. 8	8.1
Feb. 9	8.1
Feb. 10	8.1
Feb. 11	8.1
Feb. 12	8.1
Feb. 13	8.3
Feb. 14	8.2

http://waterdata.usgs.gov/nwis/rt

A Teaching Perspective

1. How does this lesson teach students to become better problem solvers?

2. How does this lesson teach students to communicate mathematics?

3. How does this lesson teach students to reason mathematically?

4. What questions should the teacher pose during this lesson?

5. What teaching suggestions would you offer this teacher?

A Learning Perspective

1. What should be the learning outcomes for this lesson?

2. What prior learning is needed for students to be successful with this lesson?

3. What evidence of learning is provided by this lesson?

4. What evidence of learning is lacking?

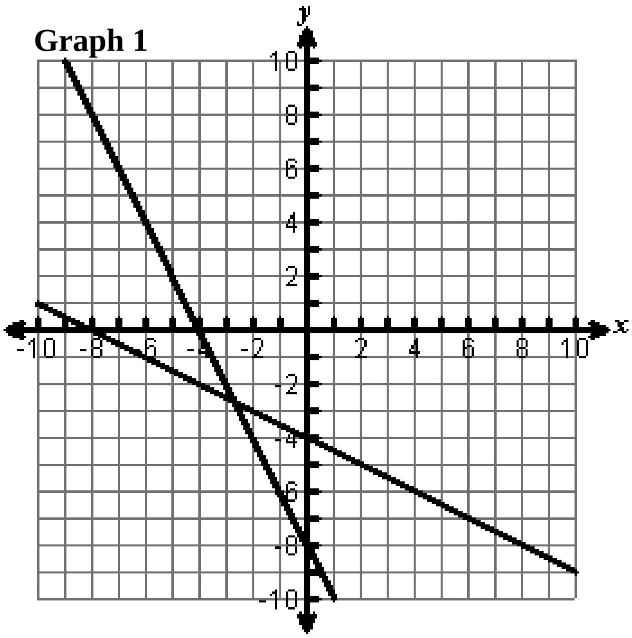
5. What misconceptions might students develop based on this lesson?

6. What about this lesson helps students learn to value mathematics?

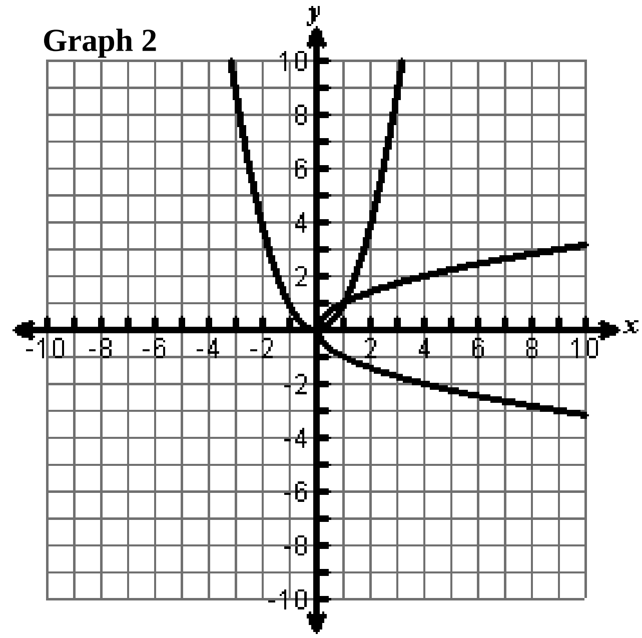
7. How does this lesson help students become more confident in their ability to do mathematics?

Know When to Fold 'Em

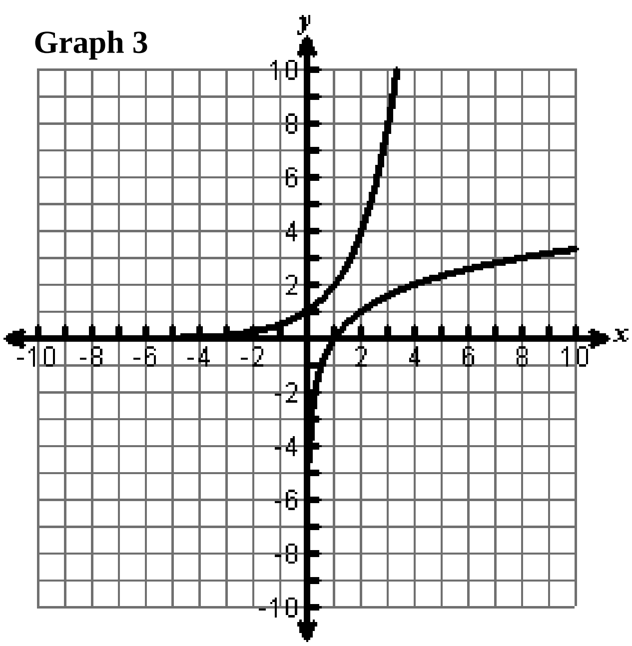
Graph 1



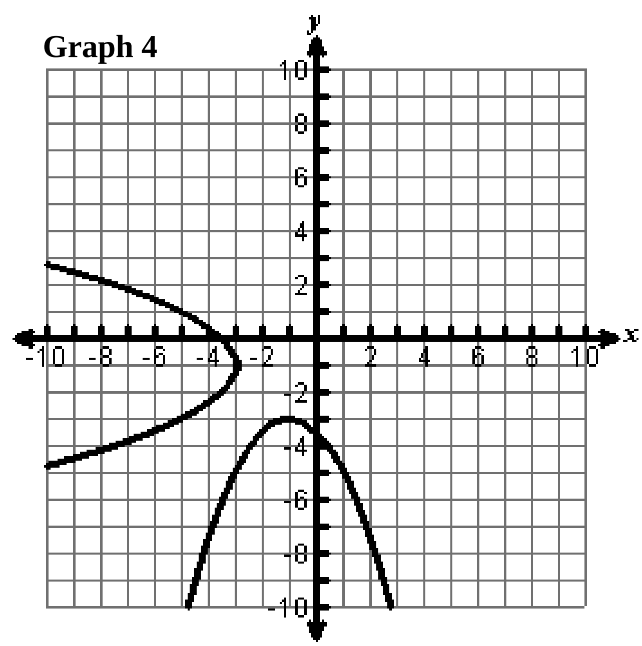
Graph 2



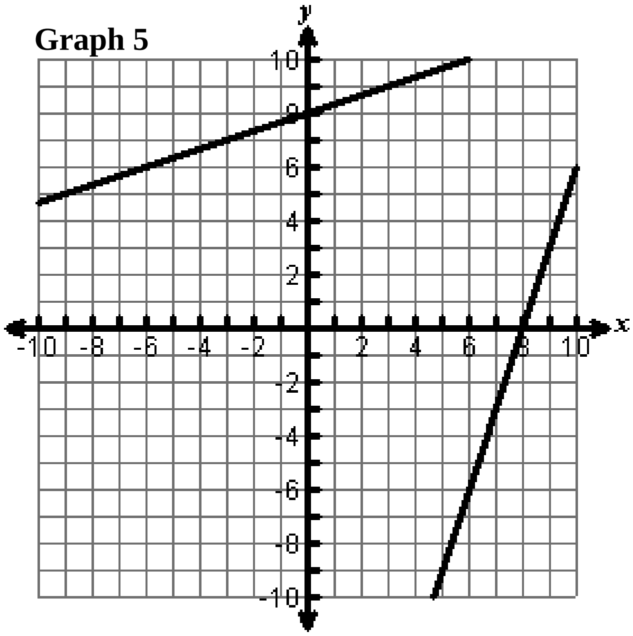
Graph 3



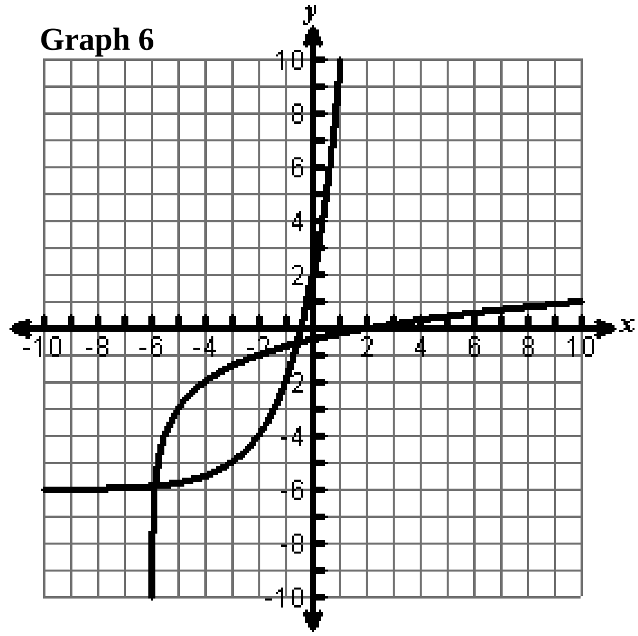
Graph 4



Graph 5



Graph 6



Vocabulary Organizer

My definition	**Personal Association**
Example	**Non-Example**

5E Student Lesson Planning Template

Description	Activity
Engage The activity should be designed to generate student interest in a problem situation and to make connections to prior knowledge. The instructor initiates this stage by asking meaningful questions, posing a problem to be solved, or by showing something intriguing.	
Explore The activity should provide students with an opportunity to become actively involved with the key concepts of the lesson through a guided exploration requiring them to probe, inquire, and question. The instructor actively monitors students as they interact with each other and the activity.	
Explain Students collaboratively begin to sequence events/facts from the investigation and communicate these findings to each other and the instructor. The instructor, acting in a facilitation role, formalizes student findings by providing further explanations and additional meaning or information, such as correct terminology.	
Elaborate Students extend, expand, or apply what they have learned in the first three stages and connect this knowledge with prior learning to deepen understanding. Instructors can use the Elaborate stage to verify students' understandings.	
Evaluate Evaluation occurs throughout students' learning experiences. More formal evaluation can be conducted at this stage. Instructors can determine whether the learner has reached the desired level of understanding the key ideas and concepts.	

Strategies that Support English Language Learners (ELL)

Strategy	Explore, Explain, Elaborate 1
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond.	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Be conscious of tone and diction. Speak slowly and distinctly.	
Incorporate language skills (reading, writing, speaking, and listening) into instruction.	

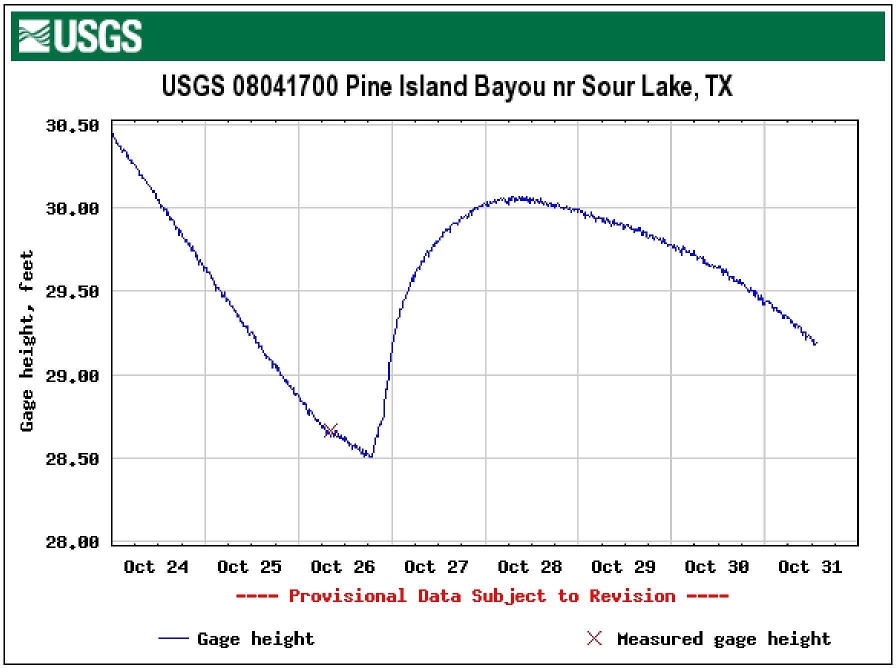
Strategies that Support Students with Special Needs

Strategy	Explore, Explain, Elaborate 1
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond.	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Use a system of quick response to needs and accommodations including progress monitoring to inform instruction.	
Accommodate materials for format, structure, sequence, etc. as needed.	

Participant Pages: Inverses of Functions

<u>Part 1</u>: Generating an Inverse Relation

During the Engage phase, you examined graphs of streams' gage heights versus time. Sometimes, hydrologists are concerned about a particular gage height and when the gage for the stream measured that height. In those cases, it makes sense to consider a plot of time versus gage height.



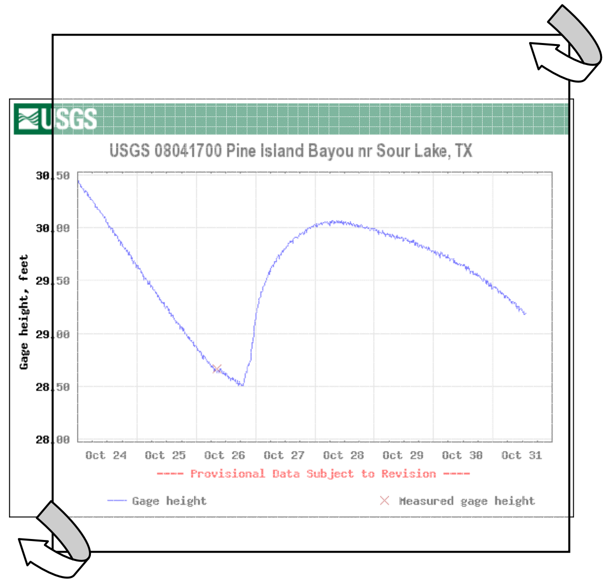
1. For the scatterplot of gage height versus time, what are the inputs and outputs?

2. Does this scatterplot represent a functional relationship? How can you tell?

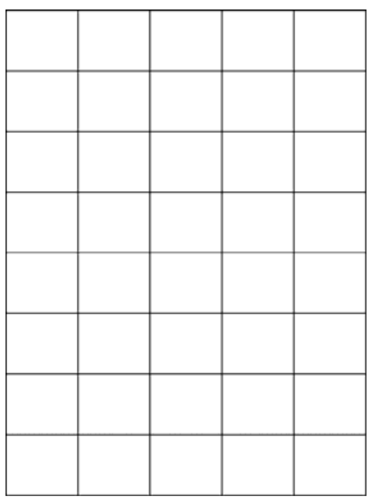
3. What would the graph of the same data look like if we plotted time versus gage height? Sketch a prediction of the graph.

Trace the graph of gage height versus time onto a sheet of patty paper. Be sure to trace the plot and the axes then label your axes on your patty paper.

Take the top-right corner in your right hand and the bottom-left corner in your left hand and flip the patty paper over. Align the origin on the patty paper with the origin of your original graph and align the axes on the patty paper with the axes on the graph.



4. Sketch the resulting graph.



5. How is the new graph similar to the original graph? How are they different?

6. Is the new graph a function? How do you know? Given the situation, does it make sense for the graph to be or not to be a function? Explain your reasoning.

On the activity sheet, **Know When to Fold 'Em**, trace each graph (both curves) onto a piece of patty paper. Be sure to also trace and label the axes.

Fold the patty paper to find all possible lines of symmetry for each pair of graphs shown. Identify the equation for each possible line of symmetry. Record the number of lines of symmetry and their equations for each graph on your recording sheet.

	Total Number of Lines of Symmetry	**Equation(s) of line(s) of symmetry**
Graph 1		
Graph 2		
Graph 3		
Graph 4		
Graph 5		
Graph 6		

7. What patterns do you notice among the lines of symmetry for each of the graphs?

8. Which transformation describes the folds across a line of symmetry for your graphs?

9. Make a summary statement describing the relationship between each pair of curves in the set of given graphs.

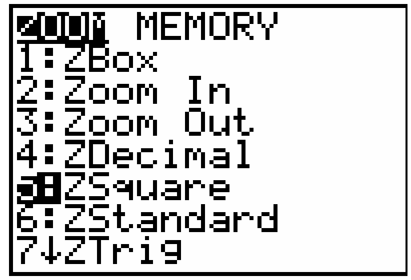
<u>Part 2</u>: Numerical and Symbolic Representations

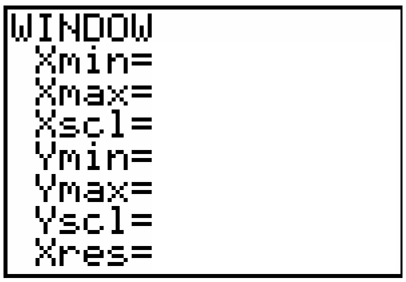
The following coordinates were used to create a geometric design for a quilting pattern.

List 1	1	1	0	0	−1	−1	−2	−1	−2	−1	1
List 2	2	4	5	4	5	4	4	3	3	2	2

1. Create a connected scatterplot of L_2 versus L_1. Sketch your graph and describe your viewing window.

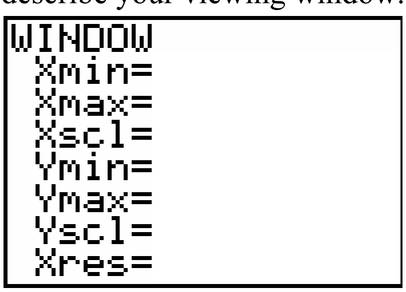
   TIP: Be sure to use a square window. After selecting your appropriate domain and range, use the Zoom-Square feature to square out the grid in your viewing window.





2. What do you think would happen to the graph if we reversed the *x*- and *y*-values?

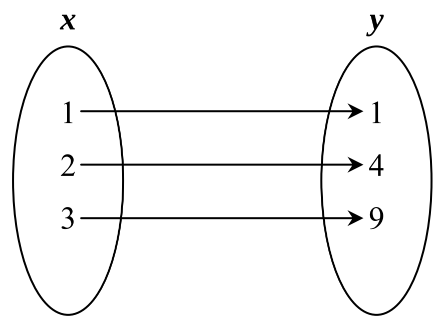
3. Create a second connected scatterplot of L_1 versus L_2. Use a different plot symbol for this scatterplot. Graph this scatterplot with your original scatterplot. Sketch your graph and describe your viewing window. You may need to re-square your viewing window.



4. Compare the two scatterplots. How are they alike? How are they different?

5. How are the x- and y-coordinates related from the first scatterplot to the second scatterplot? How could you represent this relationship symbolically?

Recall that a mapping shows how domain elements for a relation relate, or "map to" their corresponding range elements. For example, the following mapping shows how the x-values $\{1, 2, 3\}$ map to their corresponding y-values $\{1, 4, 9\}$ for the function $y = x^2$.



Enter the functions $Y_1 = 2x - 8$ and $Y_2 = \frac{1}{2}x + 4$ into your graphing calculator.

6. Use the table feature of your graphing calculator to generate values for a mapping for Y1 to show the replacement set for y when $x = \{5, 6, 7, 8, 9\}$.

7. Generate a mapping for Y2 to show the replacement set for y when $x = \{2, 4, 6, 8, 10\}$.

8. How are the two mappings related?

9. In each mapping, to how many y-values does any given x-value map? Would you expect this to be true for other domain and range elements? How do you know?

10. What does this reveal about the relationships in each mapping?

11. In each mapping, how many x-values map to any given y-value? Would you expect this to be true for other domain and range elements? How do you know?

12. What does this reveal about the relationships in each mapping?

13. Examine the graphs of Y1 and Y2. How are they related? (Hint: Be sure you are using a square viewing window.)

Enter the functions $Y_3 = \frac{2}{3}x - 7$ and $Y_4 = \frac{3}{2}x + 7$ into your graphing calculator.

14. Use the table feature of your graphing calculator to generate a mapping for Y3 to show the replacement set for y when $x = \{0, 3, 6, 9\}$.

15. Use the table feature of your graphing calculator to generate a mapping for Y4 to show the replacement set for y when $x = \{-7, -5, -3, -1\}$.

16. How are the two mappings related?

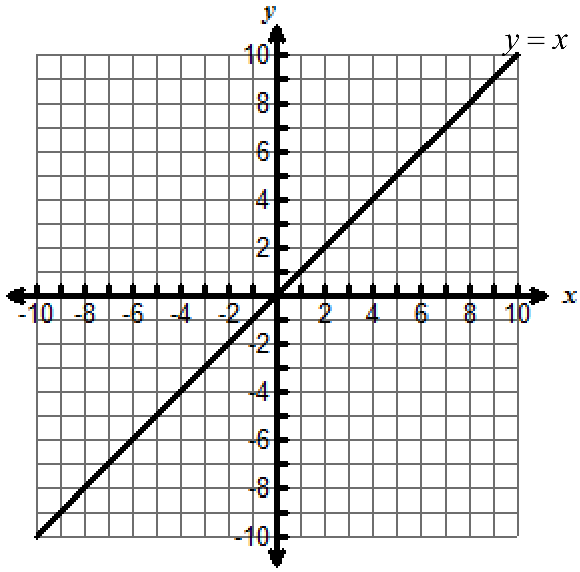
17. Examine the graphs of Y3 and Y4. How are they related? (Hint: Be sure you are using a square viewing window.)

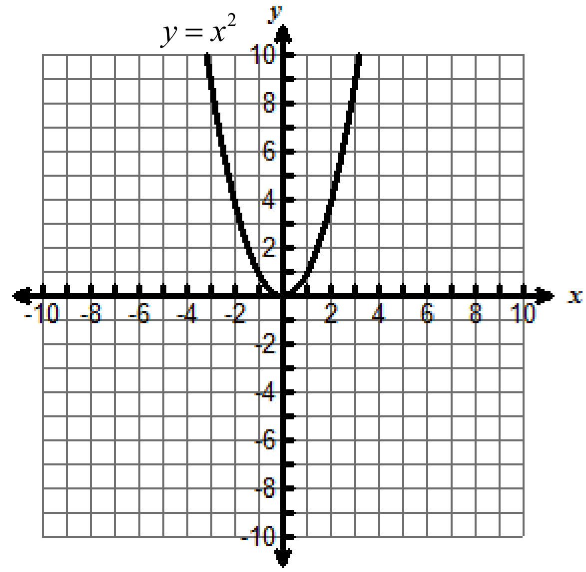
18. The functions in Y1 and Y2 are called “inverse relations” whereas the functions in Y3 and Y4 are not. Based on your mappings, graphs, and equations, why might this be the case?

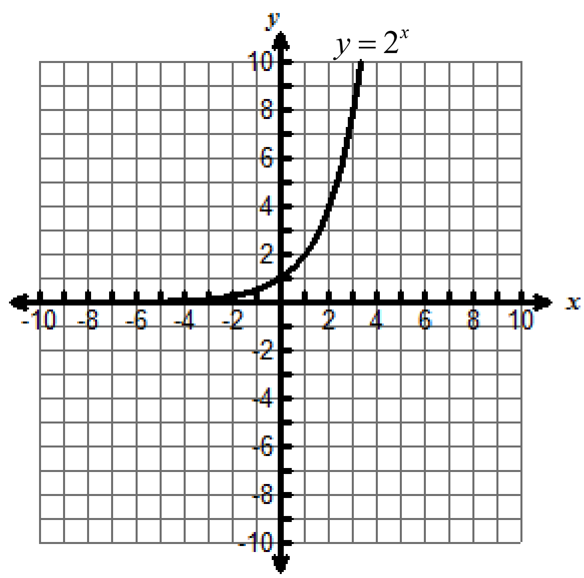
19. Based on your response to the previous question, how might we describe inverse relations graphically, numerically, and symbolically?

<u>Part 3</u>: Investigating Linear Functions

In Algebra 1, students investigate linear, quadratic, and exponential functions. The graphs of the parent functions are shown.

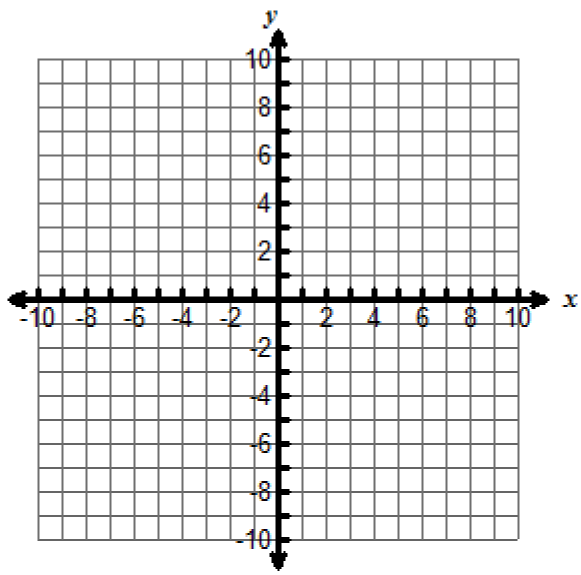






Trace each parent function onto a separate piece of patty paper. Be sure to trace and label the axes as well.

1. Reflect the linear parent function across the line $y = x$. Sketch your resulting graph.



2. What is the domain and range of the inverse of the linear parent function? How do they compare with the original function?

3. Is the inverse of the linear parent function also a function? How do you know?

4. What kind of function is the inverse of a linear function?

5. What is the inverse of the function $y = \frac{2}{5}x - 7$? Find the inverse using at least two different methods.

6. How did you determine the inverse?

7. What concepts and procedures did you apply to determine the inverse?

8. Numerically, what operations are being done to the domain values to generate the corresponding range values in the function $y = \frac{2}{5}x - 7$?

9. Numerically, what operations are being done to the domain values to generate the corresponding range values in the inverse of the function $y = \frac{2}{5}x - 7$?

10. How do these two sets of operations compare?

11. Describe the graph of the function $y = \frac{2}{5}x - 7$ in terms of transformations of the parent functions.

12. Describe the graph of the inverse of the function $y = \frac{2}{5}x - 7$ in terms of transformations of the parent functions.

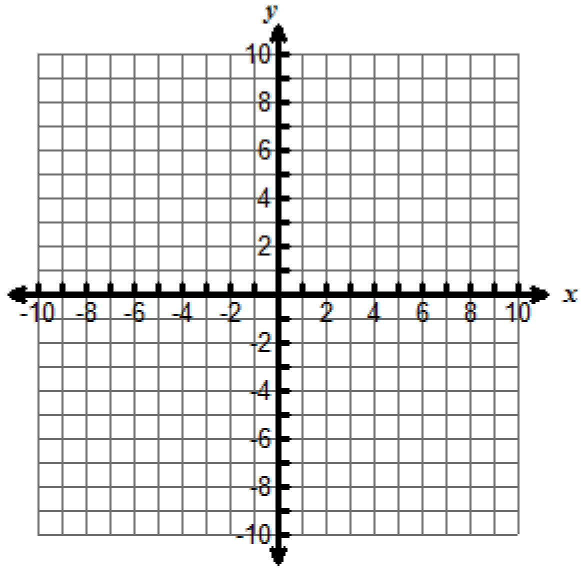
13. Compare the graphs of $y = \frac{2}{5}x - 7$, its inverse, and the line $y = x$.
    a. What do you notice about the intercepts of the graphs?

    b. Where are the three graphs concurrent? What is the significance of this point?

    c. In terms of transformations, how do the graphs of the original function and its inverse compare?

Part 4: Quadratic Functions

1. Reflect the quadratic parent function across the line $y = x$. Sketch your resulting graph.



2. What is the domain and range of the inverse of the quadratic parent function? How do they compare with the original function?

3. Is the inverse of the quadratic parent function also a function? How do you know?

4. If the inverse is not a function, how can we restrict the domain and/or range of the original function so that the inverse is also a function?

5. What kind of function is the inverse (range restricted)of a quadratic function?

6. Is either the original parent function or its range restricted inverse one-to-one? How do you know?

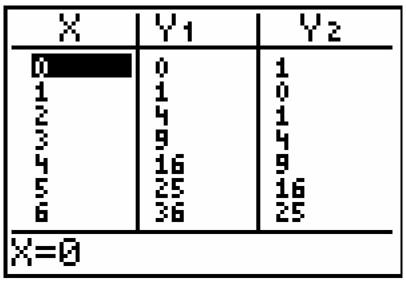
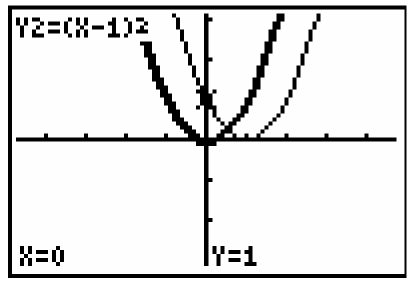
7. What is the inverse function of the function $y=-3(x-1)^2+2$? Find the inverse using at least two different methods.

8. How did you determine the inverse?

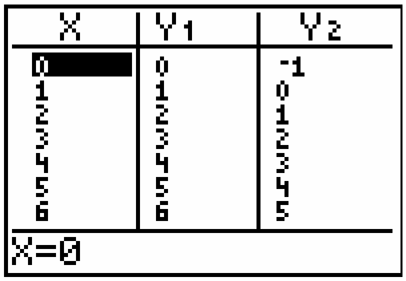
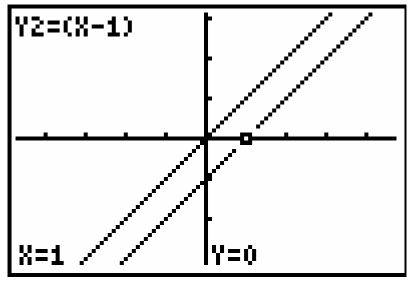
9. What concepts and procedures did you apply to determine the inverse?

10. Numerically, what operations are being done to the domain values to generate the corresponding range values in the function $y=-3(x-1)^2+2$?

11. From the perspectives of number operations and transformations, develop the quadratic function, $y=-3(x-1)^2+2$, from the parent function $y = x^2$. Include tabular, graphical, and symbolic representations of the number operation as it applies to the function.

Number Operation	**Tabular**	**Graphical**	**Symbolic**
Subtract 1 from the x-value in the parent function $y = x^2$.			$y=(x-1)^2$

12. Now, use number operations to describe what happens to the domain elements, represented by the variable *x*, as you develop the quadratic function, $y = -3(x-1)^2 + 2$, from $y = x$.

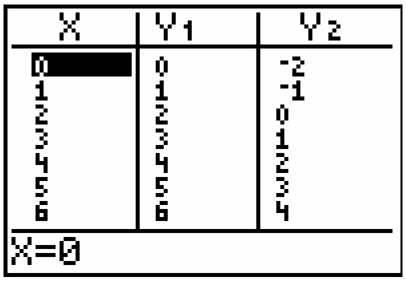
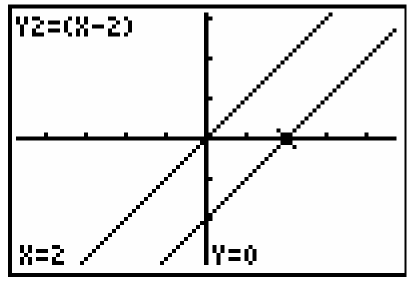
Number Operation	Tabular	Graphical	Symbolic
Subtract 1 from the parent function $y = x$			$y = x - 1$

13. How does each successive number operation transform the function numerically, graphically, and symbolically?

14. Numerically, what operations are being done to the domain values to generate the corresponding range values in the inverse of the function $y = -3(x-1)^2 + 2$?

15. How does this set of operations compare to the operations applied to generate the function $y = -3(x-1)^2 + 2$?

16. Use number operations to describe what happens to the domain elements, represented by the variable x, as you develop the square root inverse of the function, $y = -3(x-1)^2 + 2$, from $y = x$.

Number Operation	Tabular	Graphical	Symbolic
Subtract 2 from the parent function $y = x$.			$y = x - 2$

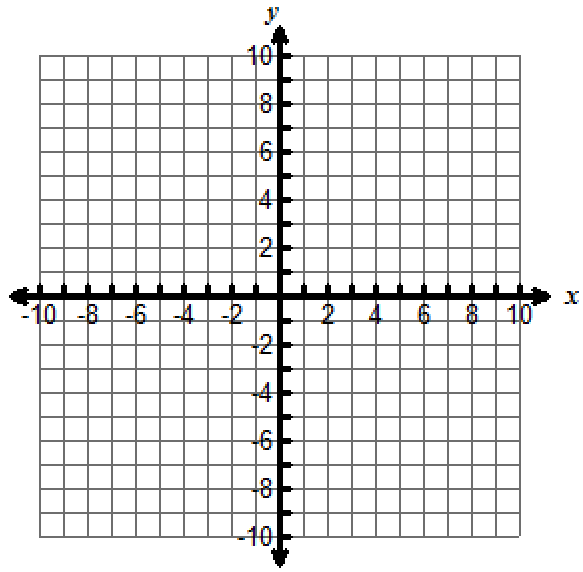
17. How do the inverse operations relate to the inverse function?

18. Generalize how inverse relations for quadratic functions compare to their corresponding original functions. Consider each of the four representations and use the table below to record your responses.

Number Operation	**Tabular**	**Graphical**	**Symbolic**

<u>Part 5:</u> Exponential Functions

1. Reflect the exponential parent function, $y = 2^x$, across the line $y = x$. Sketch your resulting graph.



2. What is the domain and range of $y = 2^x$?

3. What is the domain and range of the inverse of $y = 2^x$? How do they compare with the original function?

4. What asymptote(s) does the original function, $y = 2^x$ have? Why does this asymptote exist?

5. What asymptote(s) does the inverse of $y = 2^x$ have? How do they compare to the asymptotes of the original function?

6. Is the inverse of $y = 2^x$ also a function? How do you know?

7. What kind of function is the inverse of an exponential function?

8. Is either the original parent function or its inverse one-to-one? How do you know?

9. What is the inverse of the function $y = \frac{1}{2}(10)^{x-1} + 3$? Find the inverse using at least two different methods.

10. How did you determine the inverse?

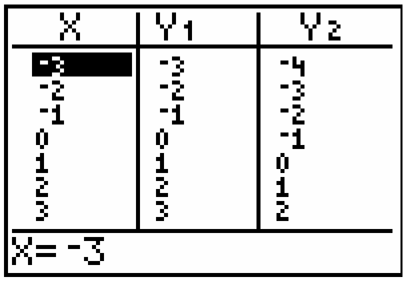
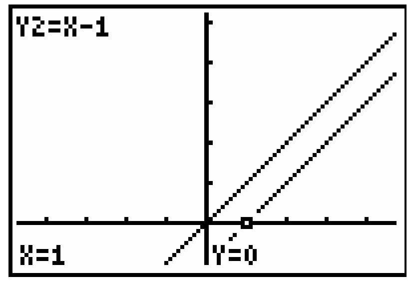
11. What concepts and procedures did you apply to determine the inverse?

12. Numerically, what operations are being done to the domain values to generate the corresponding range values in the function $y = \frac{1}{2}(10)^{x-1} + 3$?

13. Numerically, what operations are being done to the domain values to generate the corresponding range values in the inverse of the function $y = \frac{1}{2}(10)^{x-1} + 3$?

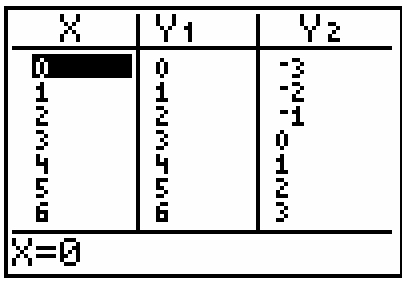
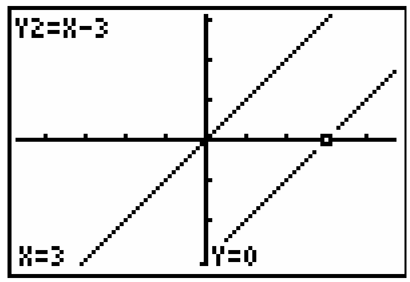
14. How do these two sets of operations compare?

15. Use number operations to describe what happens to the domain elements, represented by the variable *x*, as you develop the exponential function, $y = \frac{1}{2}(10)^{x-1} + 3$, from $y = x$.

Number Operation	Tabular	Graphical	Symbolic
Subtract 1 from the parent function $y = x$			$y = x - 1$

16. How does each successive number operation transform the function numerically, graphically, and symbolically?

17. Use number operations to describe what happens to the domain elements, represented by the variable *x*, as you develop the logarithm inverse of the function, $y = \frac{1}{2}(10)^{x-1} + 3$, from $y = x$.

Number Operation	Tabular	Graphical	Symbolic
Subtract 3 from the parent function $y = x$			$y = x - 3$

18. How do the inverse operations relate to the inverse function?

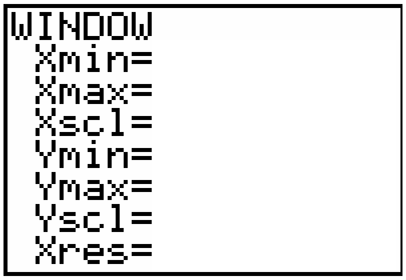
19. Generalize how inverse relations for exponential functions compare to their corresponding original functions. Consider each of the four representations and use the table below to record your responses.

Number Operation	Tabular	Graphical	Symbolic

<u>Part 6</u>: Compositions of Functions

Enter the function $y = 2x - 1$ into *Y1* of your graphing calculator's function editor. Enter the inverse of this function into *Y2*. Enter the composition of *Y2* and *Y1* into *Y3*.

1. Sketch the graphs of the three functions (describe your viewing window). What relationships and patterns do you notice?



2. Look at the table values for each of the three functions. What do you notice?

3. How could you represent the composition of these two functions with a mapping?

4. What effect does composition of inverse functions have? Why do you think this is so?

5. Does the order of composition matter? Explain your answer.

6. Investigate the composition of a quadratic function such as $y = x^2 - 2$ and its inverse <u>function</u>. Describe the result numerically, graphically, and symbolically.

<u>Part 7</u>: Extension

1. Investigate the composition of a quadratic function such as $y = x^2 - 2$ and its inverse <u>relation</u>. Describe the result numerically, graphically, and symbolically.
   Case 1: Inverse is not a function – Composition of Inverse(Original)

   Case 2: Inverse is not a function – Composition of Original(Inverse)

2. Based on your experiences with linear and quadratic functions, what would you expect to be true about compositions of other types of functions, such as exponential, rational, or polynomial functions? Give examples or counterexamples.

Vocabulary Organizer

My definition	**Personal Association**
Example	**Non-Example**

5E Student Lesson Planning Template

Description	Activity
Engage The activity should be designed to generate student interest in a problem situation and to make connections to prior knowledge. The instructor initiates this stage by asking meaningful questions, posing a problem to be solved, or by showing something intriguing.	
Explore The activity should provide students with an opportunity to become actively involved with the key concepts of the lesson through a guided exploration requiring them to probe, inquire, and question. The instructor actively monitors students as they interact with each other and the activity.	
Explain Students collaboratively begin to sequence events/facts from the investigation and communicate these findings to each other and the instructor. The instructor, acting in a facilitation role, formalizes student findings by providing further explanations and additional meaning or information, such as correct terminology.	
Elaborate Students extend, expand, or apply what they have learned in the first three stages and connect this knowledge with prior learning to deepen understanding. Instructors can use the Elaborate stage to verify students' understandings.	
Evaluate Evaluation occurs throughout students' learning experiences. More formal evaluation can be conducted at this stage. Instructors can determine whether the learner has reached the desired level of understanding the key ideas and concepts.	

Strategies that Support English Language Learners (ELL)

Strategy	**Explore, Explain, Elaborate 2**
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Be conscious of tone and diction. Speak slowly and distinctly.	
Incorporate language skills (reading, writing, speaking, and listening) into instruction.	

Strategies that Support Students with Special Needs

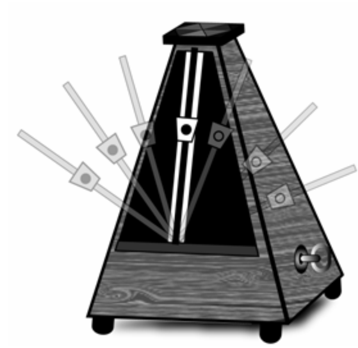
Strategy	Explore, Explain, Elaborate 2
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Use a system of quick response to needs and accommodations including progress monitoring to inform instruction.	
Accommodate materials for format, structure, sequence, etc. as needed.	

Participant Pages: Square Root Functions

Explore

<u>Part 1</u>: Setting the Stage

A metronome frequently is used in music to mark exact time using a repeated tick. The frequency of the ticks varies in musical terms from slow *(largo,* about 40-60 beats per minute) to fast *(presto,* 168-208 beats per minute). Individual instrumentalists, choirs, bands, and orchestras all use metronomes to ensure that the beat of the music is consistent with the instructions of the composer and does not unintentionally speed up or slow down while the piece is being played.



What do you notice about the frequency of the ticks and setting of the weight as the music is being played?

<u>Part 2</u>: Modeling and Gathering Data

Divide participants into groups of 4. Each person in the group has a job.

Materials manager: Gets the necessary materials, directs the team in setting up the investigation, holds the pencil with the suspended bottle, and shortens the spring when needed.

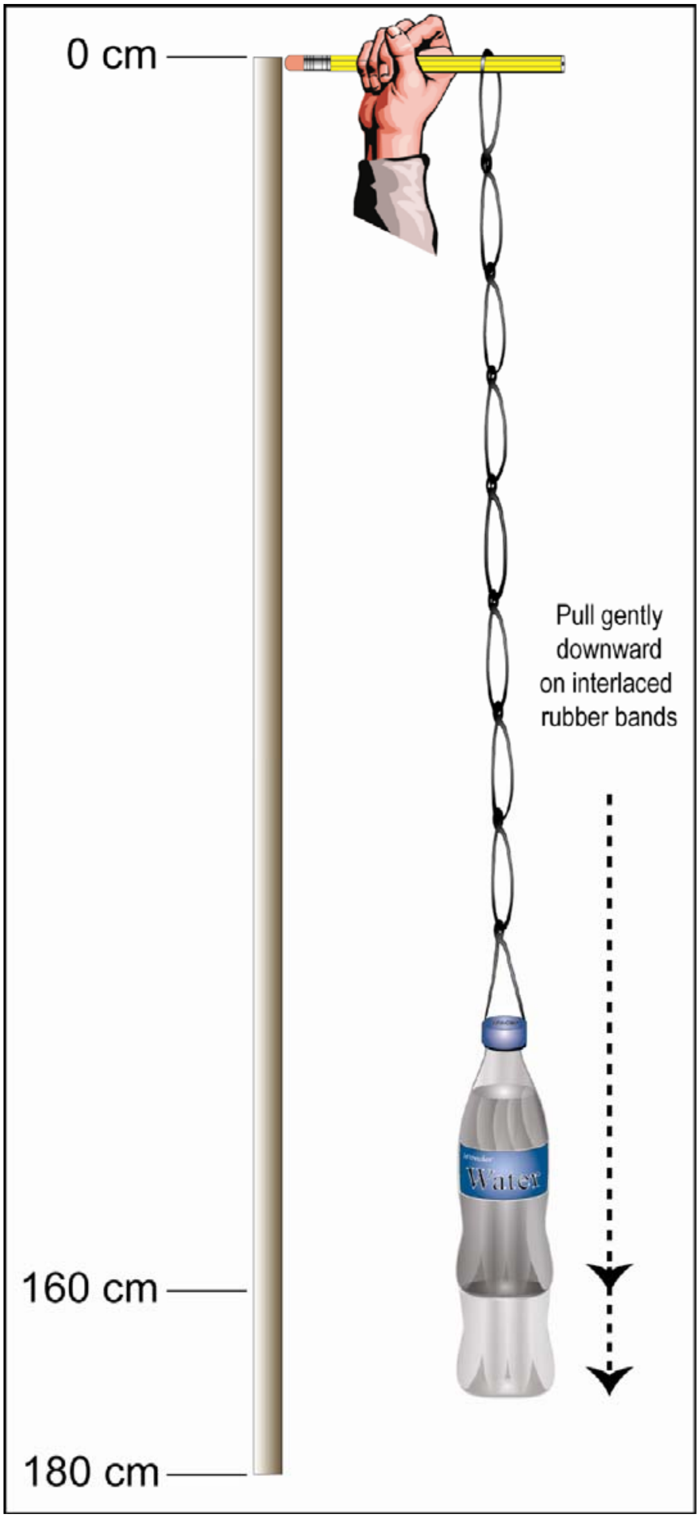
Measures manager: Measures the distance from the pencil to the bottom of the bottle for each length, initiates the bounce by pulling the suspended bottle down an additional 10 centimeters and counts the bounces (10 at each height).

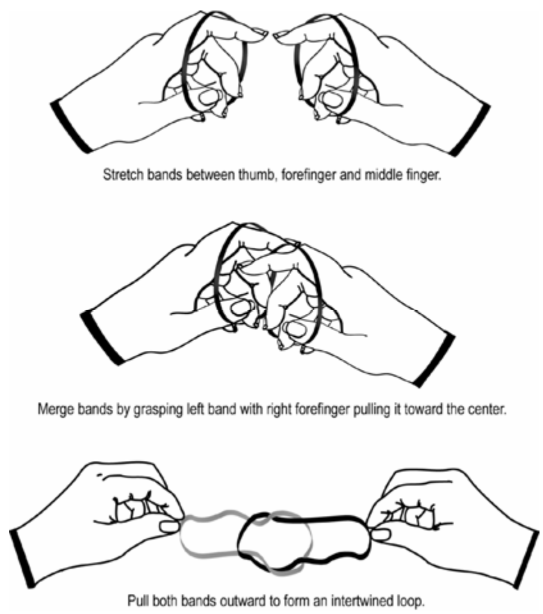
Time manager: Uses a stop watch to determine the length of each 10-bounce period of time. The time starts when the bottle is released by the measures manager and ends when the bottle completes its 10^{th} bounce.

Data manager: Records the necessary measurements in the table and shares the data with the team.

Set-up Instructions

Step 1. The materials manager should get the necessary materials and ask two of the team members to secure the tape measure or meter sticks against the wall. The tape measure or meter sticks should be positioned perpendicular to the floor so that the "zero end" is at 180-200 centimeters above the floor.





Step 2. While the tape measure or meter sticks are being positioned, the materials manager and remaining team member(s) build the rubber band spring by looping rubber bands together until the length of the spring is about 1 meter. This task will go more quickly if each person makes about half of the spring. Then the pieces can be joined.

Step 3. Secure one end of the rubber band spring to the pencil and the other around the neck of the bottle. It also works to remove the cap, insert the end of the spring in the bottle, and screw the cap back on.

Step 4. The measures manager secures the spring so that it is approximately 160 centimeters in length. After the bottle remains motionless for a few seconds, he should measure the actual length of the spring. The length of the spring includes the length of the rubber band and the length of the bottle.

Step 5. The measures manager pulls the bottle downward about 10 cm and releases it.

Step 6. The time manager starts the stopwatch when it is released and stops it at the end of 10 complete bounces. It may be helpful to have all team members count aloud together.

Step 7. The data manager records the number of seconds in the table under Trial 1 for 160 cm. *Hint: It is more meaningful to start with the spring fully extended and to shorten the spring than to begin at the top and work down.*

Step 8. Repeat for Trials 2 and 3. Average the data from the 3 trials and record in the Average Time column.

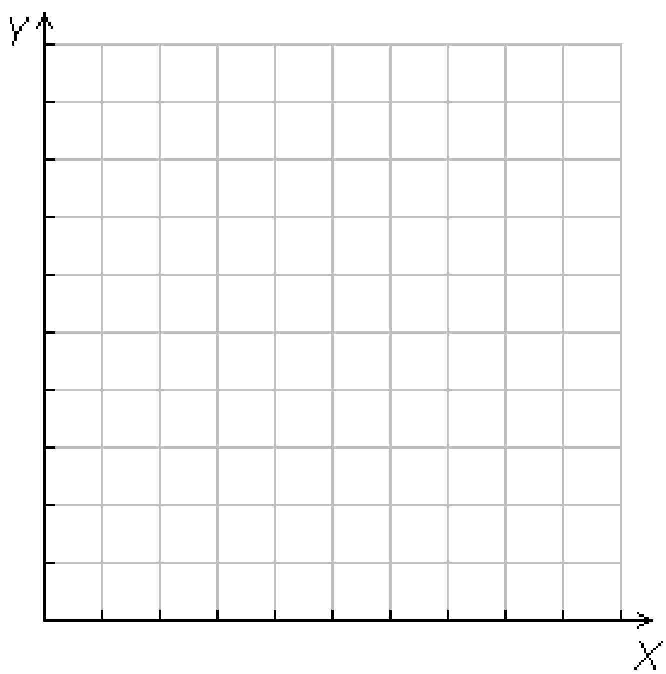
Step 9. The materials manager who is holding the pencil shortens the spring by wrapping it around the pencil until the desired length of 140 is obtained.

Step 10. Continue repeating the procedure with shortened lengths of rubber band spring. Continue to record your data.

1. Record your data in the table below.

Approximate Length of Spring (cm) x	Actual Length of Spring (cm) x	Trial 1	Trial 2	Trial 3	Average Time y
0					
20					
30					
40					
60					
80					
100					
120					
140					
160					

2. Make a scatterplot of the data you collected.



3. What is the independent variable?

4. What is the dependent variable?

5. Write a dependency statement relating the two variables.

6. What is a reasonable domain for the set of data?

7. What is a reasonable range for the set of data?

8. Is this data set continuous or discrete? Why?

9. Does the set of data represent a function? Why?

10. Write a summary statement about what happened in this data investigation.

11. Is the function increasing or decreasing?

12. Is the rate of change constant?

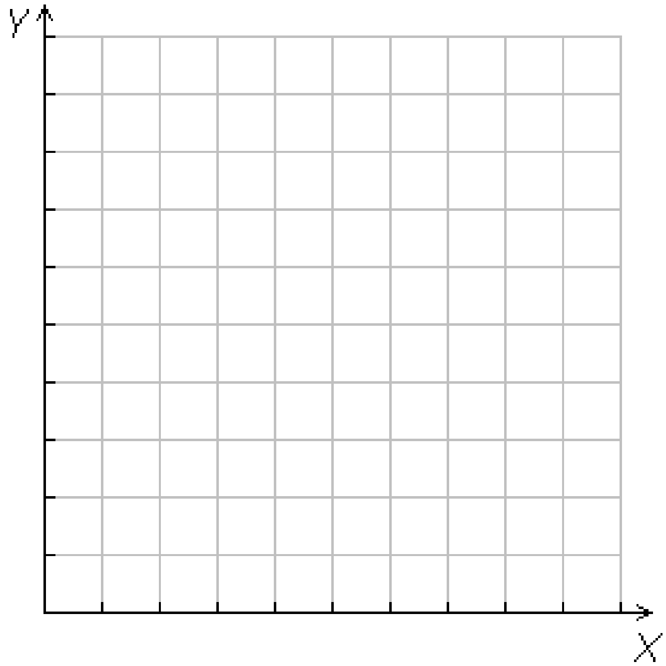
13. Does the data you collected appear to be a linear, quadratic, exponential, or some other type of parent function? Why?

14. How is the bottle bounce activity similar to the ticks of a metronome?

15. What kind of function do you think models the ticking of the metronome? Why?

<u>Part 3</u>: Analyzing the Data

1. How can you determine whether this function is the inverse of another parent function using patty paper?



2. Input your values into L_1 and L_2 of a graphing calculator, letting L_1 be independent values and L_2 dependent values, and create a scatterplot of the original data. Sketch your graph.

3. Create a second scatterplot that represents an inverse of the data. Use a different plot symbol for this scatterplot. Determine a new domain and range, and set a new viewing window. Sketch your graph.

4. What changes must you make to the window to view the second set of data?

5. Which parent function do the *reflected* points most closely appear to represent?

6. How did you determine your function?

7. How might your confirm your conjecture?

8. Without using regression, find a function that approximates your data for Plot 2.

9. Does your viewing window allow you to see both sides of the parabola? If not, readjust your viewing window. Sketch your graph.

10. How could you use this function to find a function that would approximate the first scatterplot you graphed?

11. Reset your window to view Plot 1. Enter the equation in the equation editor. Is your graph a close fit to the data in Plot 1? Sketch your graph.

12. Compare and contrast the graphs of a quadratic function and a square root function. How are they similar, and how are they different?

13. Why are there no negative coordinates in the square root function?

14. What is the domain of a square root function?

15. What is the range of a square root function?

16. What conclusions can you make about the attributes of a square root function?

17. What conclusions can you make about the collected data?

<u>Part 4</u>: Making Symbolic Generalizations

Transformations of Square Root Functions Card Sort

1. Place the cards in the proper row and column.

Description	Example	Example	Notation
Vertical Translation Up			
Vertical Translation Down			
Horizontal Translation Left			
Horizontal Translation Right			
Vertical Stretch			
Vertical Compression			
Reflection			

2. Describe the role of *a*.

3. Describe the role of *h*.

4. Describe the role of *k*.

5. Using *x, a, h*, and *k*, write an equation that could be used to summarize the transformations to the square root function.

6. Revisiting the bottle bounce investigation, describe the transformation to the square root parent function that represents your data.

<u>Part 5 (Optional Extension)</u>: Investigating the Coefficient of *x*.

1. Using $f(x) = a\sqrt{\frac{1}{b}(x-h)} + k$, predict the changes in the parent function for the following functions. Then check with your graphing calculator.

   a. $f(g) = \sqrt{-x}$

   b. $f(g) = \sqrt{-3x} + 4$

   c. $f(g) = \sqrt{\frac{1}{2}(x-3)}$

   d. $f(g) = 2\sqrt{-\frac{1}{3}(x+4)} - 5$

2. What can you summarize about transformations of the square root parent function as a result of changes to $\frac{1}{b}$?

<u>Part 6 (Optional Extionsion)</u>: Connecting the Roles of *a*, *h*, and *k* in Square Root and Quadratic Functions

Equation 1 $$y = 3\sqrt{x-5} + 6$$

Equation 2 $$y = 7(x-2)^2 + 4$$

1. Find the inverse of Equation 1.

2. Numerically and graphically compare and contrast Equation 1 and its inverse.

3. Find the inverse of Equation 2.

4. Numerically and graphically compare and contrast Equation 2 and its inverse.

5. Find the inverse of $y = a\sqrt{(x-h)} + k$.

6. Find the inverse of $y = a(x-h)^2 + k$.

7. Summarize the relationship between h and k in the square root transformation form and h and k in the quadratic transformation (vertex) form.

8. Summarize the relationship between a in the square root transformation form and a in the quadratic transformation (vertex) form.

<u>Part 7</u>: Solving Square Root Equations and Inequalities

1. Consider the system of equations $y = \sqrt{x+3}$ and $y = 4$.

   a) Graph the system and sketch the graph. What are the domain and range for each function in this system?

   b) How can you determine the solution to this system of equations graphically or tabularly?

   c) What are the coordinates of the point that is a solution for this system?

   d) How can you use the transitive property to write this system as one equation?

   e) How can you solve this equation algebraically?

   f) What would be your solution set if you had been given $\sqrt{x+3} \geq 4$?

2. Consider the system of equations $y = \sqrt{x+2}$ and $y = x$.

   a) Graph the system and sketch the graph. What are the domain and range for each function in this system?

   b) How can you determine the solution to this system of equations graphically or tabularly?

   c) What are the coordinates of the point that is a solution for this system?

   d) How can you use the transitive property to write this system as one equation?

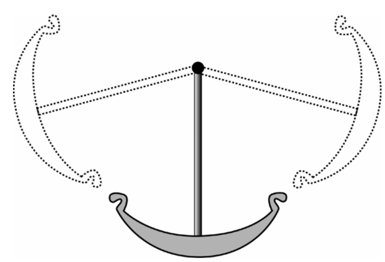
   e) How can you solve this equation algebraically?

   f) Are both solutions valid? Why or why not?

3. Choose one of the following problems. Work with your group to find the solutions(s). Justify your answer. Use chart paper to display your work.

   A. Jim is an accident investigator who was asked to determine whether a driver's excessive speed was a factor in a traffic accident. The traditional equation used to determine the speed at which a vehicle was traveling at the onset of the skid is $V_s = \sqrt{2aS_s}$, where a is the deceleration force of gravity times friction, and S_s is the length of the skid marks. If the speed limit is 60 mph and the skid marks are 225 ft. long, was the driver exceeding the speed limit? (Use 6.01 for a.) What is the maximum length skid mark that would have exonerated the driver? Justify your answer.

   B. At Thalia's favorite amusement park, there is a ride called the "Pirate Ship". People sit in what looks like a huge ship. The "ship" then swings back and forth. Thalia notices that it takes somewhere between 7 and 8 seconds for the ride to make one complete swing back and forth. What is the minimum and maximum length of the swinging bar?

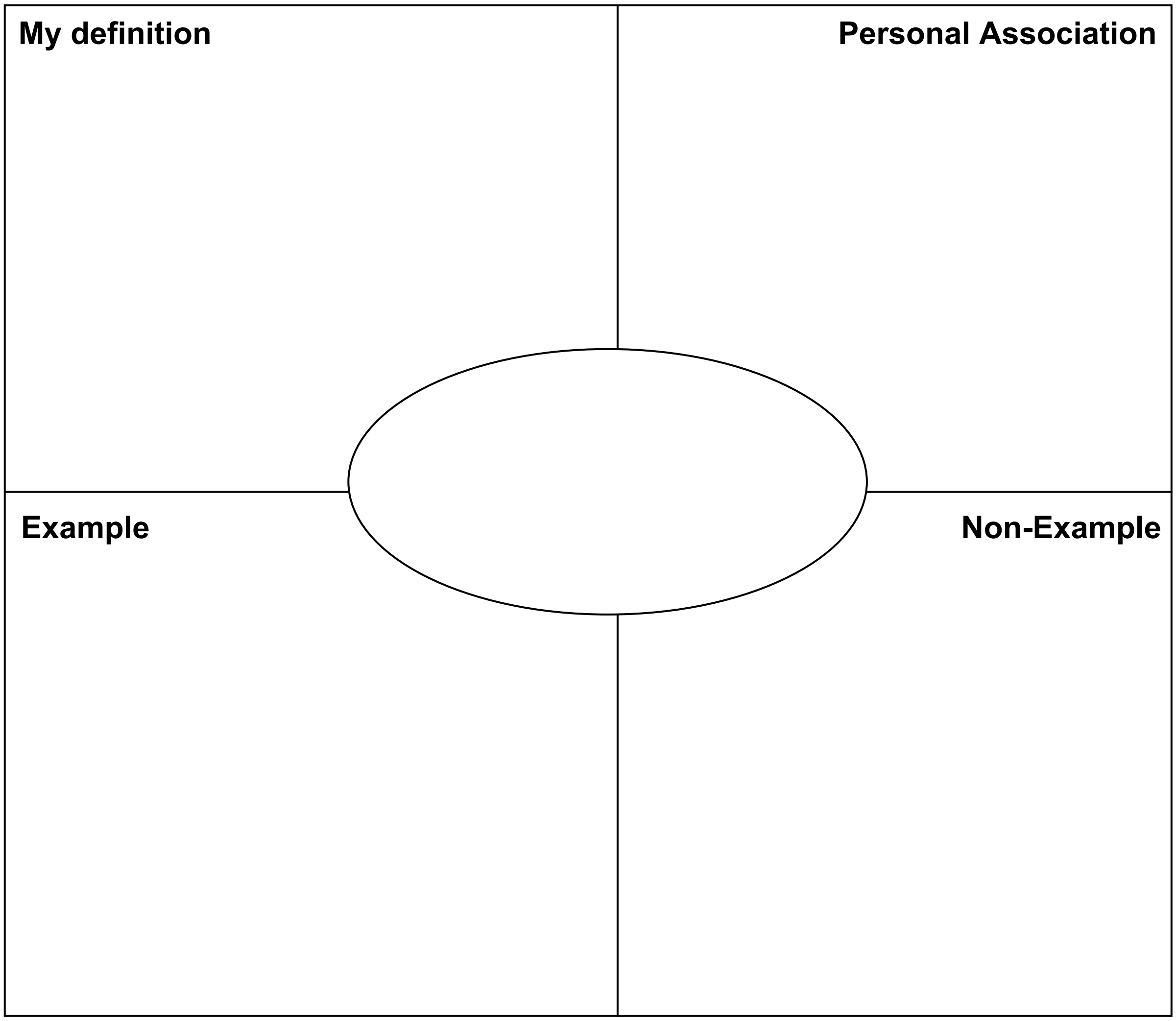


The function that represents the time in seconds of one complete swing, t, based on the height of the swinging bar, h, in feet, is $t = 2\pi\sqrt{\frac{h}{32}}$.

C. Arnie was taking a picture from the window of his apartment. Unfortunately, he dropped the camera, which landed on the ground at least 2 seconds later. The equation that models the time, t, it takes for an object to fall h meters is $t = \sqrt{\frac{2h}{9.81}}$. From what height did Arnie drop the camera?

D. Sharon's mother bought a grandfather clock and asked Sharon to determine how long the pendulum must be so the clock keeps accurate time. Sharon found the formula $t = 2\pi\sqrt{\frac{L}{g}}$, where t is the time for one complete swing of the clock pendulum, L is the length of the pendulum, and g is acceleration due to gravity (which is 980 cm/sec^2). Since the time for a complete swing of the pendulum of a grandfather clock must be 2 seconds, how long should the pendulum be?

Vocabulary Organizer



5E Lesson Planning Template

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Strategies that Support English Language Learners (ELL)

Strategy	Explore, Explain, Elaborate 3
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Be conscious of tone and diction. Speak slowly and distinctly.	
Incorporate language skills (reading, writing, speaking, and listening) into instruction.	

Strategies that Support Students with Special Needs

Strategy	Explore, Explain, Elaborate 3
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Prompt student to compare and contrast concepts, procedures, and generalizations.	
Use a system of quick response to needs and accommodations including progress monitoring to inform instruction.	
Accommodate materials for format, structure, sequence, etc. as needed.	

Participant Pages: The Fire Station Problem

A fire station is located on Main Street and has buildings at every block to the right and to the left. You will investigate the relationship between the address number on a building and its distance in feet from the fire station. On average, a mile in the city is composed of 16 city blocks. So each city block is about 330 feet long (5280 feet ÷ 16 = 330 feet). Each building in centered on the block.

1. Complete the table below that relates the address of a building (x) with its distance in feet from the fire station (y).

Address Number (x)	**Distance in Feet from the Fire Station (y)**

2. Which building is 660 feet way from the fire station? Explain your answer.

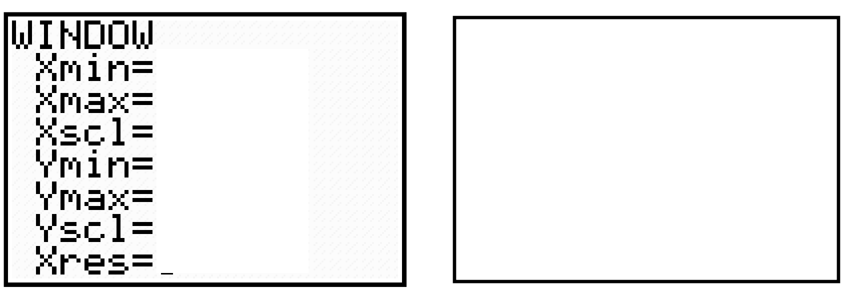
3. If we send someone to the building that is 660 feet away from the fire station, how will she know that she has arrived at the correct place?

4. What words might we use to describe two locations that are the same distance from the fire station?

5. Suppose the buildings on Main Street are renumbered as if they are on a number line so that the location of the fire station represents 0. How do we describe two numbers with the same distance from 0?

6. Draw a scatterplot that represents the data in the table.

7. Make a scatterplot of your data using your graphing calculator. Describe your viewing window.

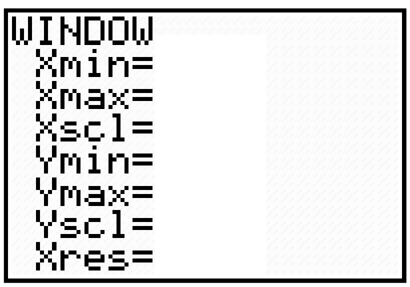


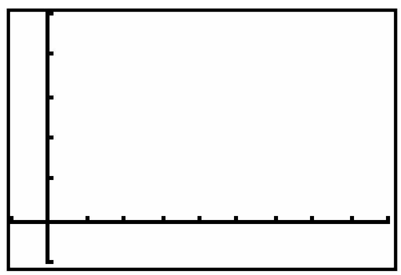
8. What function or functions might students use to describe the scatterplot?

9. Find two linear functions that pass through the data points. What process did you use to find the equations of the lines?

10. Graph the equations on your calculator. How are the equations similar? How are they different?

11. If necessary adjust the window to clearly see the intersection of the two lines. What does the intersection of these two lines represent? Sketch the graph.





12. Where do the equations fit the graph of the data points? Where do the equations not fit the graph of the data points?

13. How well do the linear equations model the data points?

14. Write summary statements about conceptual understanding of the absolute value of a number and linear equations that model a situation using absolute value.

Participant Pages: The Relay Race

Pretend you are in an unusual relay race. The object of the race is to walk toward the CBR at a slow steady rate as if to pick up something and then to walk backwards away from the CBR at the same rate without stopping. The person whose rate walking towards the CBR matches the rate walking away from the CBR and who changes direction instantly wins the race!

1. Predict and sketch the distance versus time graph of the volunteer's walk in the space below.

2. Using the Ranger program on the APPS menu of the calculator, a CBR and a link cable, collect data on the relay race. You may want to move to an area that provides room for you to walk.

3. What is the shape of the graph of your walk? How does the graph of the walk compare to your prediction?

4. How many times is the walker a given distance from the CBR?

5. At what point on the graph does the direction of the walk change? How can you interpret this point in terms of the time and distance?

6. What part of the graph represents your motion toward the CBR? What function rule best describes the walk toward the CBR?

7. What is the domain for this part of the walk? How does this domain compare to the domain of the function?

8. What part of the graph represents motion away from the CBR? What function rule best describes the walk away from the CBR?

9. What is the domain for this part of the walk? How does this domain compare to the domain of the function?

10. How do the functions compare? Does this match your expectation? If there are differences, what might explain them?

11. How can we write one function rule that describes the entire walk?

12. How does this function remedy domain restrictions we encountered by using two linear functions?

13. What is the parent function for absolute value?

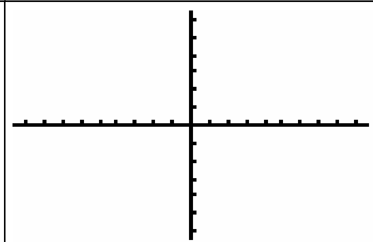
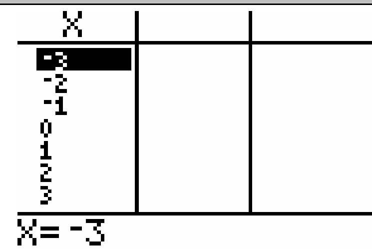
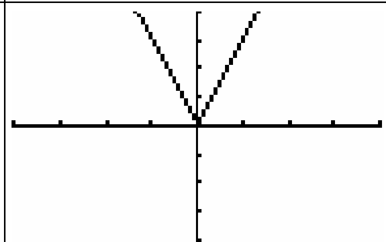
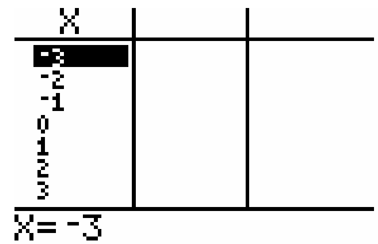
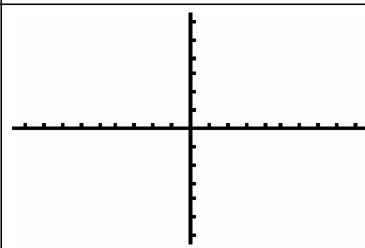
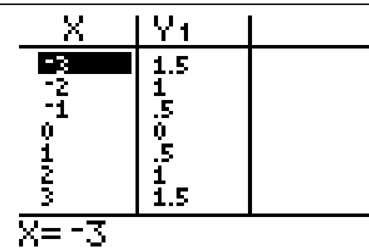
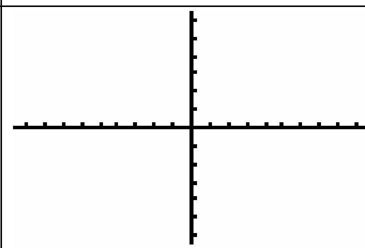
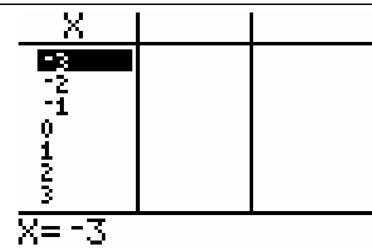
14. What are the characteristics of absolute value functions?

15. How can we use what we know about the linear functions we wrote to write one absolute value function that fits the graph of the walk? Explain your answer.

.

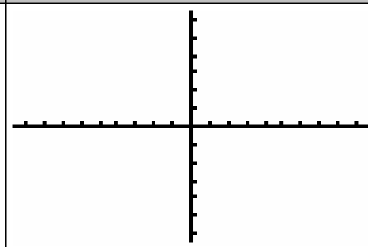
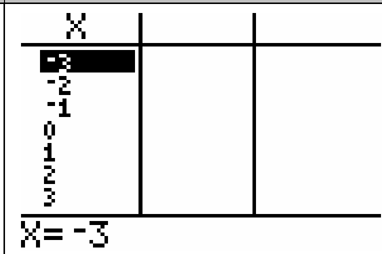
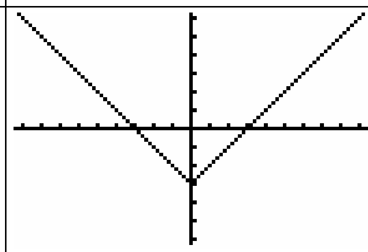
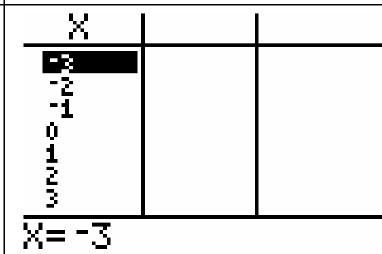
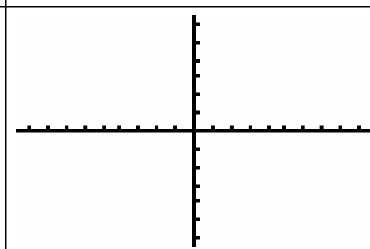
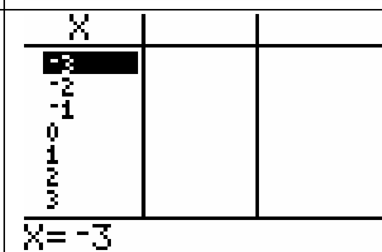
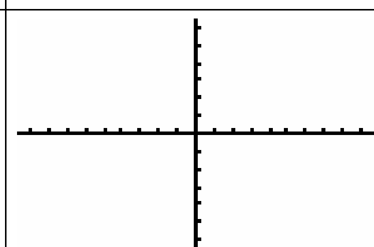
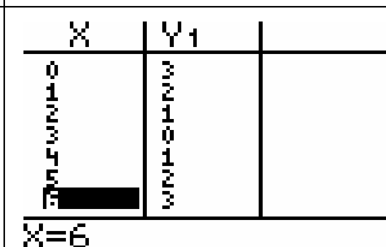
16. Write a summary statement about how modeling an absolute value function through an activity such as The Relay Race connects real-life situations to Algebra II concepts.

Participant Pages: Transformations of the Parent Function: Changes to a

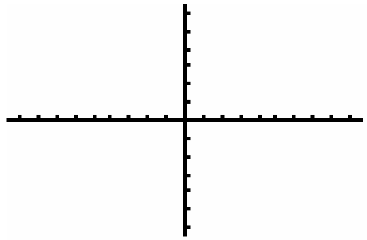
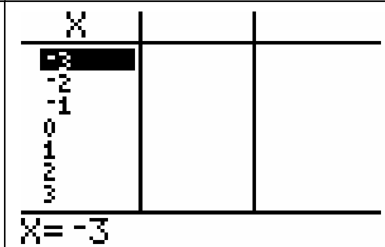
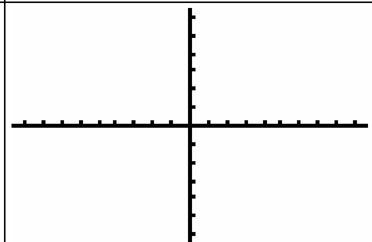
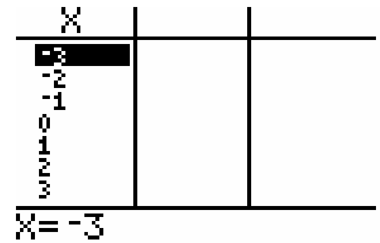
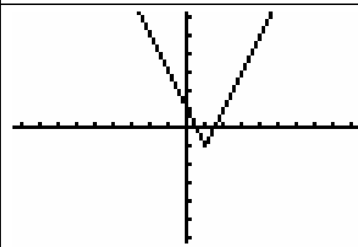
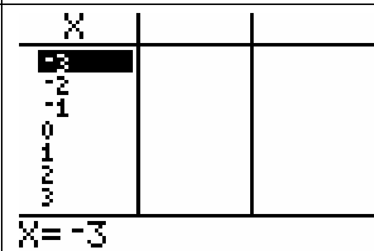
Function	Graph	Table	Effect on Parent Function
$y = \|x\|$			
			
			
			The y-values of the parent function have been multiplied by a factor of –2, which vertically stretches the graph of the parent function and reflects it across the x-axis.

Write a generalization about how the parameter *a* affects the graph of the parent function.

Participant Pages: Transformations on the Parent Function: Changes to *h* and *k*

Function	Graph	Table	Effect on Parent Function
$y = \|x\| + 2$			
			
			The x-coordinate of the vertex is moved to the solution for $x + 2 = 0$. The graph of the parent function is shifted to the left by 2 units.
			
Write a generalization about how changes in the parameters *h* and *k* affect the graph of the parent function.			

Participant Pages: Transformations to the Parent Functions: Changes to *a*, *h*, and *k*

<table>
<tr><th>Equation</th><th>Graph</th><th>Table</th><th>Effect on Parent Function</th></tr>
<tr><td>$y = |x - 1| + 2$</td><td></td><td>
</td><td></td></tr>
<tr><td></td><td></td><td>
</td><td>The vertex of the parent function has been translated to (–2, –4).</td></tr>
<tr><td></td><td></td><td>
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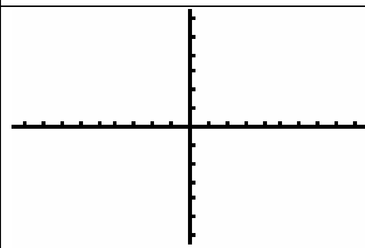
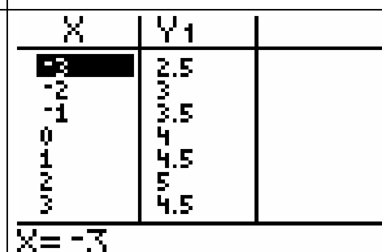
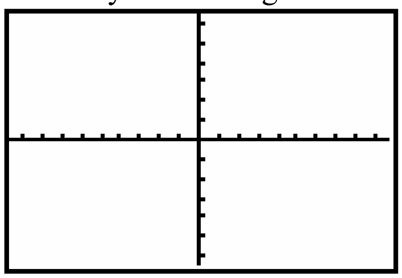
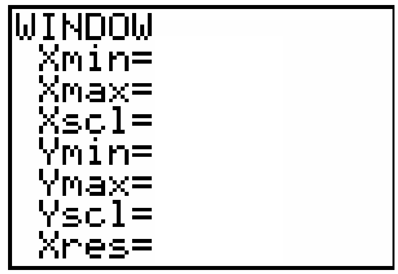
<tr><td></td><td></td><td>
</td><td></td></tr>
<tr><td colspan="4">Write a generalization about how changes to the parameters a, h, and k affect the graph of the parent function.</td></tr>
</table>

Participant Pages: Solving Absolute Value Equations and Inequalities

1. Consider the system of equations $y = |x+3|$ and $y = 4$. Graph the system and sketch the graph. Describe your viewing window.

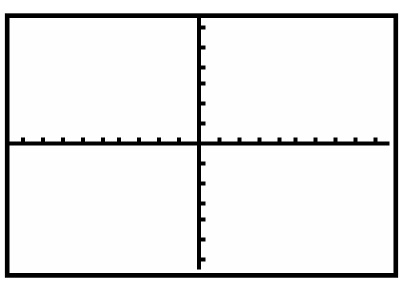
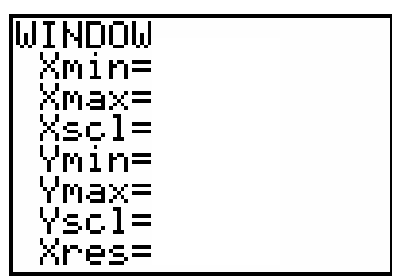



   a) What are the domain and range of each function in this system?

   b) What are the coordinates of the points that are solutions for this system? Why are there two solutions?

   c) How can you use the concept of substitution to write this system as one equation?

2. Graph the functions $y = x+3$, $y = -(x+3)$ and $y = 4$ and sketch the graph. Describe your viewing window.

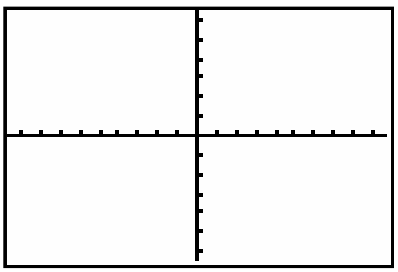



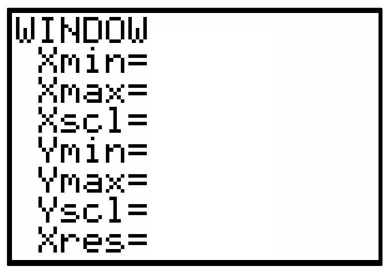
   a) What are the domain and range for each function in this system? How does this system of equations compare to the original system?

b) What are the coordinates of the points that are solutions for this system? How do these solutions compare to the solutions of the original system?

c) How can you use the concept of substitution to write this system of three equations as a system of two equations?

3. Graph the functions $y = x + 3$, $y = 4$, and $y = -4$. Graph this system and sketch the graph. Describe your viewing window.





a) What are the domain and range for each function in this system?

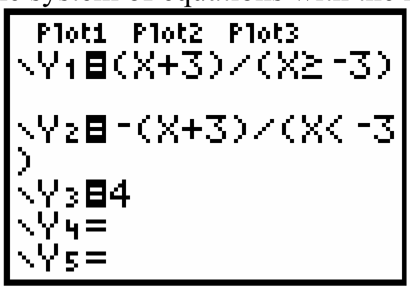
b) What are the coordinates of the points that are solutions for this system? How do the solutions for the graph above compare to the original solutions in question 1?

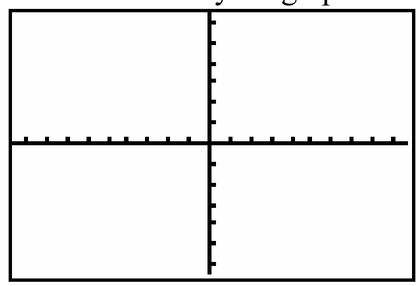
c) Compare the graph above and the graph in question 2 to the graph of the original system. Which graph is conceptually related to the graph of the original system? Which graph is not conceptually related? Why?

d) What misconceptions might arise by setting up a process to solve $x + 3 = 4$ and $x + 3 = -4$?

e) What restrictions do we need to place on the domains of the functions $y = x + 3$ and $y = -(x+3)$ so that their graphs match the graph of the function $y = |x+3|$?

f) Graph the system of equations with the restrictions. Sketch your graph.

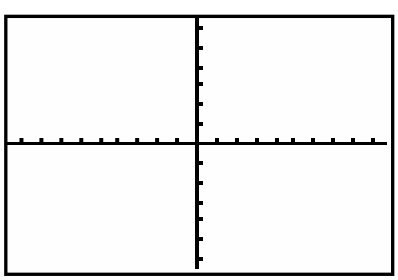


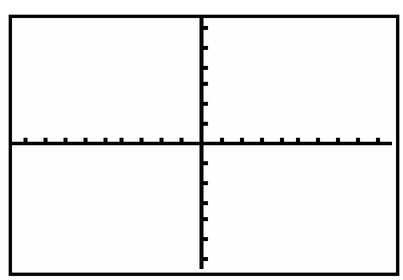


g) How does the graph of this system of equations and its solutions compare to the graph and solutions of $y = |x-1|$ and $y = 2$ in question 4a?

4. Write a system of three equations that conceptually relate to the system of equations $|x-1| = 2$.

a) Graph the system you wrote and compare it to the graph of equations $y = |x-1|$ and $y = 2$. How do the graphs compare? How do the solutions compare?

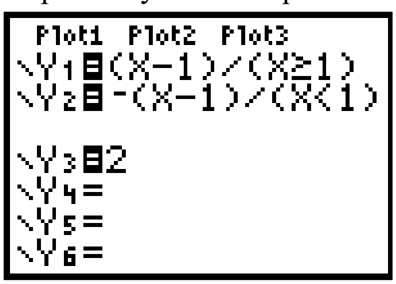


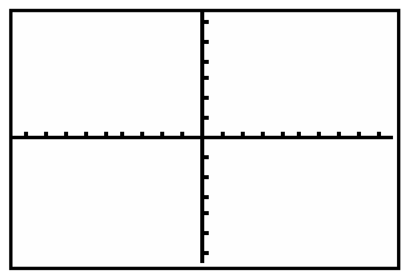


b) How do the equations $x-1=2$ and $-(x-1)=2$ relate to the graphs of the above systems?

c) What restrictions should we place on the domains of the functions in your system so that the graph of your system matches the graph of $y=|x-1|$?

d) Graph the system of equations with the restrictions. Sketch your graph.

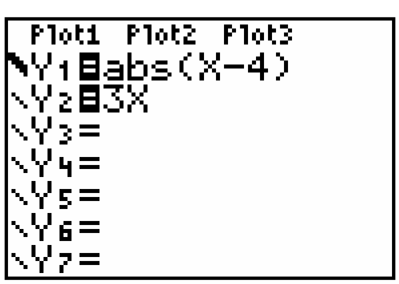


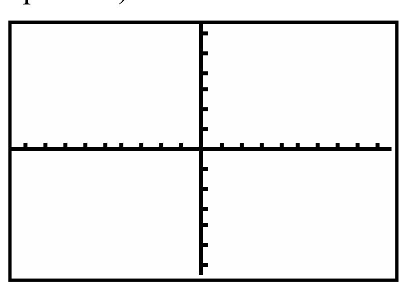


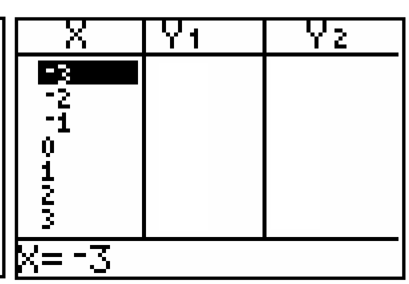
e) How does the graph of this system and its solutions compare to the graph of $y=|x-1|$ and $y=2$ in question 4a?

5. Write a statement about the connection between a system of equations such as $|x+2|=5$ and the system $x+2=5$ and $-(x+2)=5$.

6. Consider the system of equations represented by the equation $|x-4|=3x$. Graph the equations $y=|x-4|$ and $y=3x$, sketch the graph, and complete the table. (Hint: You may want to bold the absolute value equation.)

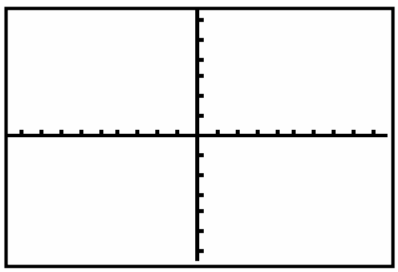
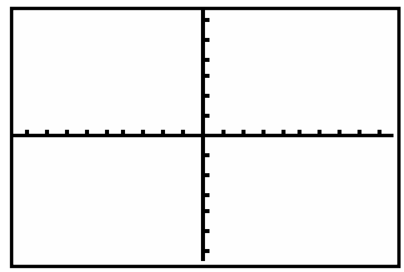






a) How many solutions does this system have? Why?

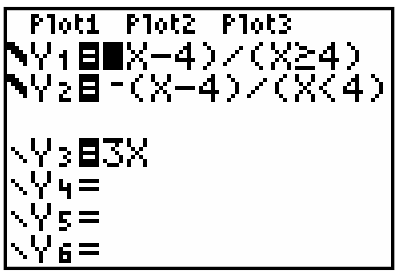
b) Compare the following systems in the table graphically, tabularly, and algebraically. (Hint: you may want to bold the equation(s) representing the absolute value function.)

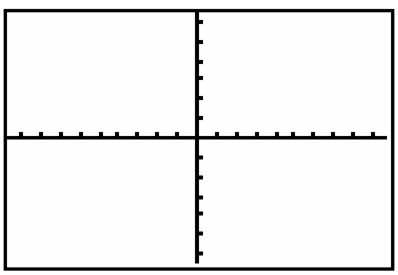
<table>
<tr><th>System A</th><th>System B</th></tr>
<tr><td>$y = x - 4$
$y = -(x - 4)$
$y = 3x$</td><td>$y = x - 4$
$y = 3x$
$y = -3x$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>
<table>
<tr><th>x</th><th>y_1</th><th>y_2</th><th>y_3</th></tr>
<tr><td>–3</td><td></td><td></td><td></td></tr>
<tr><td>–2</td><td></td><td></td><td></td></tr>
<tr><td>–1</td><td></td><td></td><td></td></tr>
<tr><td>0</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td></tr>
</table>
</td><td>
<table>
<tr><th>x</th><th>y_1</th><th>y_2</th><th>y_3</th></tr>
<tr><td>–3</td><td></td><td></td><td></td></tr>
<tr><td>–2</td><td></td><td></td><td></td></tr>
<tr><td>–1</td><td></td><td></td><td></td></tr>
<tr><td>0</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td></tr>
</table>
</td></tr>
<tr><td></td><td></td></tr>
</table>

c) What are the solutions for System A? What are the solutions for System B? How do those solutions compare to the original system?

d) Which system, A or B, conceptually relates to the original system?

e) Restrict the domains for the functions in System A, graph the new system and complete the table below. Find the table values for the solutions.





x	y_1	y_2	y_3
–3			
–2			
–1			
0			
1			
2			
3			

f) How are the solutions for System A without the restricted domain and System A with the restricted domain alike and different?

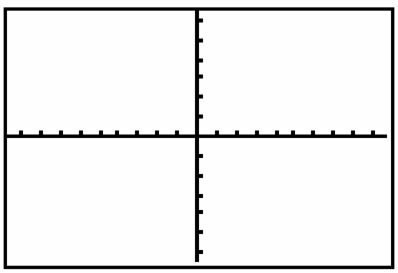
g) Why does System A without the restricted domain produce two solutions? Which solution of A is not a solution of the original system? What do we call that solution?

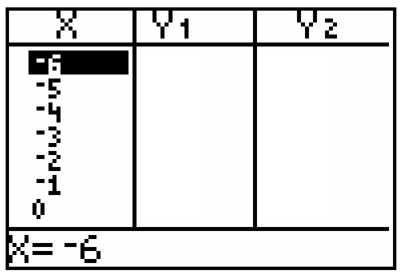
7. What understanding does graphing a system involving absolute value equations provide with regard to the actual number of solutions to the system and the corresponding equations that intersect? How does the graphical solution connect to the algebraic process?

8. Write a statement comparing the common algebraic process for solving absolute value equations to the conceptual understanding of the solutions to a system of absolute value equations.

9. Consider the system $|x+3|>4$.

   a) Show the solution graphically, tabularly, and symbolically.





   b) When we ask students to show us where $|x+3|>4$, what are we asking?

Vocabulary Organizer

My definition	**Personal Association**
Example	**Non-Example**

5E Student Lesson Planning Template

Description	Activity
Engage The activity should be designed to generate student interest in a problem situation and to make connections to prior knowledge. The instructor initiates this stage by asking meaningful questions, posing a problem to be solved, or by showing something intriguing.	
Explore The activity should provide students with an opportunity to become actively involved with the key concepts of the lesson through a guided exploration requiring them to probe, inquire, and question. The instructor actively monitors students as they interact with each other and the activity.	
Explain Students collaboratively begin to sequence events/facts from the investigation and communicate these findings to each other and the instructor. The instructor, acting in a facilitation role, formalizes student findings by providing further explanations and additional meaning or information, such as correct terminology.	
Elaborate Students extend, expand, or apply what they have learned in the first three stages and connect this knowledge with prior learning to deepen understanding. Instructors can use the Elaborate stage to verify students' understandings.	
Evaluate Evaluation occurs throughout students' learning experiences. More formal evaluation can be conducted at this stage. Instructors can determine whether the learner has reached the desired level of understanding the key ideas and concepts.	

Strategies that Support English Language Learners (ELL)

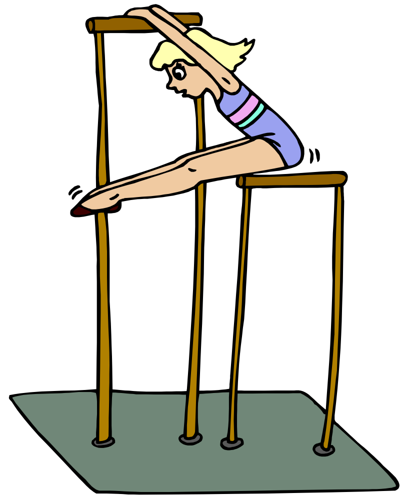
Strategy	Explore, Explain, Elaborate 4
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond.	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Be conscious of tone and diction. Speak slowly and distinctly.	
Incorporate language skills (reading, writing, speaking, and listening) into instruction.	

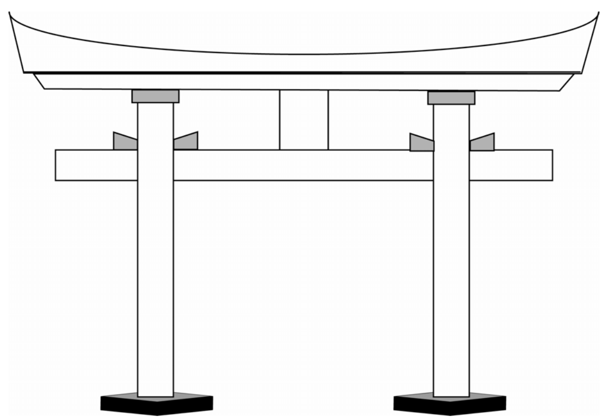
Strategies that Support Students with Special Needs

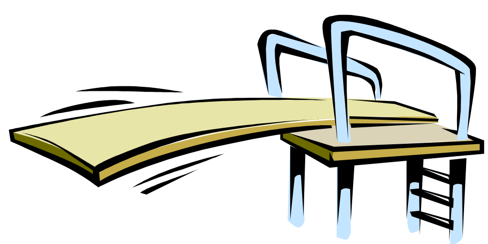
Strategy	**Explore, Explain, Elaborate 4**
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond.	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Use a system of quick response to needs and accommodations including progress monitoring to inform instruction.	
Accommodate materials for format, structure, sequence, etc. as needed.	

Participant Pages: Rational Functions

How are the objects below alike? How are they different?







<u>Part 1</u>: Linguine Cantilever

A cantilever is a projecting structure that is secured at only one end and carries a load on the other end. Diving boards and airplane wings are examples of horizontal cantilevers. Flagpoles and chimneys are vertical cantilevers. One of the most famous examples of a cantilever in architecture, which is shown below, is the Frank Lloyd Wright designed home, Fallingwater. The strength of a cantilever can be affected by variables such as length, load, cross sectional area, temperature, or elasticity. In this activity, you will be investigating the relationship between the thickness of a cantilever and the deflection in the cantilever when weight is added at the end.

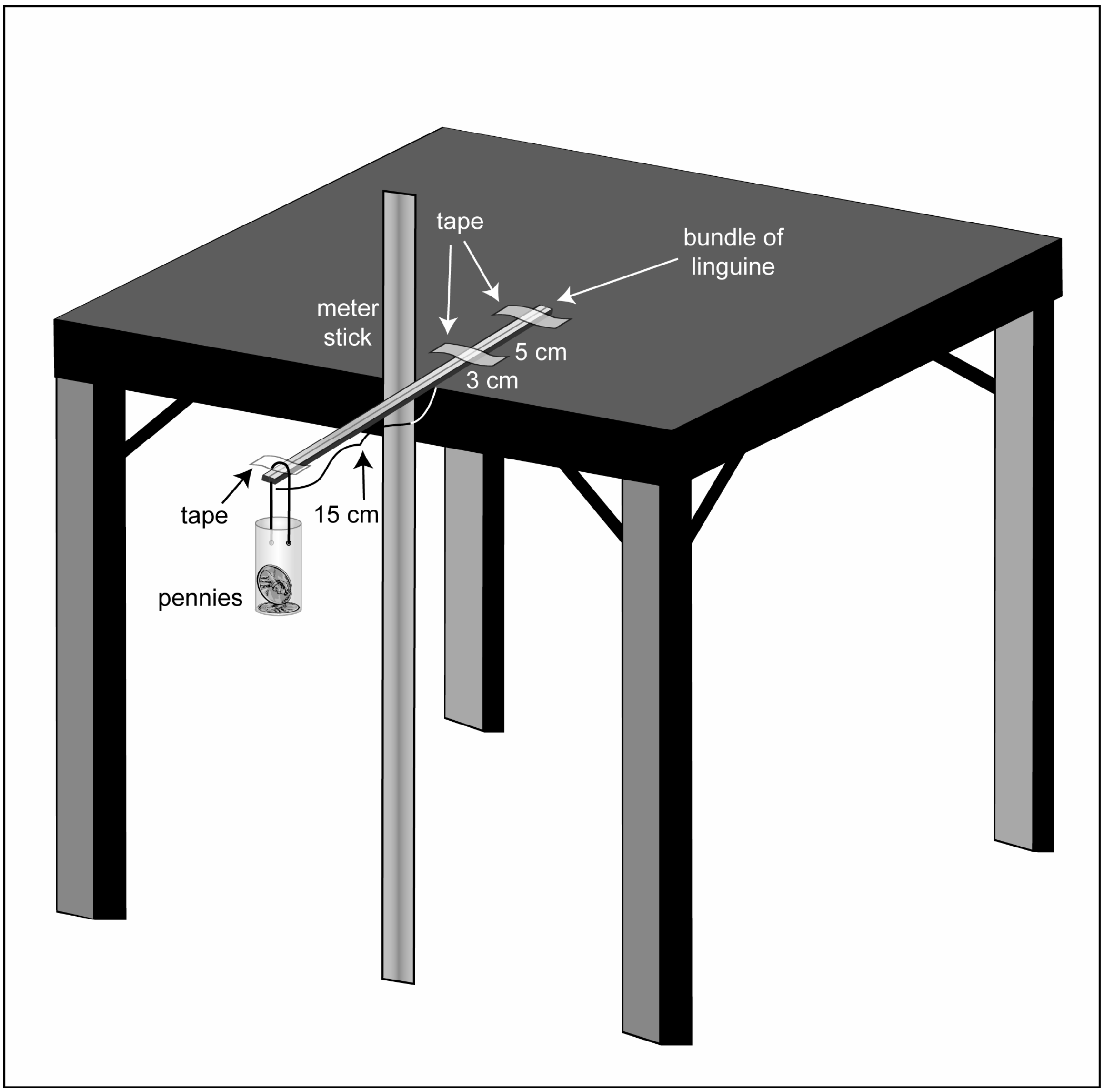
In this investigation, you will keep the length of a piece of linguine that is hanging over the edge of a desk constant as you collect data on how much the linguine deflects. Deflection is the amount that the linguine bends in the downward direction. The number of pieces of linguine will change. Since you want to keep all variables (except for the ones you are investigating) constant make sure to pay attention to the hints listed with the instructions.

Have participants get into groups of three. Each person in the group has a job.

Materials manager: Get the necessary materials, direct the team in setting up the investigation

Measures manager: Measure the amount of deflection as the investigation proceeds

Data manager: Record the necessary measurements in the table, share the data with the team



Data Collection Set-up Instructions

Step 1. The materials person should get the necessary materials and begin to make bundles of linguine. Each bundle of 1, 2, 3, 4, 5, 6, 7, and 8 pieces of linguine should be taped one inch from each end. Since linguine is not all exactly the same length, try to keep one end of the bundle lined up.

Step 2. Tape a short piece of string to the 35mm film canister to form a handle. If you do not have a film canister, use a baggie, transparent tape, and a paper clip to build a weight to hang from the linguine.

Step 3. Tape one piece of linguine with 15 centimeters hanging over the edge of a desk. Put one piece of tape approximately 3 cm from the edge of the table. Place a second piece of tape over the end of linguine. Place the load (film canister) on the end of the linguine that is hanging over the edge of the desk. Slowly place pennies in the film canister until the linguine breaks. Wait 15 seconds before adding an additional penny. Use one less penny than the number required to break one piece of linguine as the load in your bucket for the remainder of this data collection experiment.

Step 4. Tape a meter stick perpendicularly to the floor next to a desk.

Step 5. Measure the linguine's height above the floor without the film canister attached. (Hint: It is easier to consistently measure the height using the bottom of the linguine.)

Step 6. Place your pennies into the bucket. (Hint: Place the pennies gently, throwing pennies into the bucket will alter the results.)

Step 7. Place the bucket on the end of the linguine that is hanging over the edge of the desk. (Hint: Place the string at the same point on the linguine for each trial. Use a piece of masking tape to hold the bucket onto the linguine.)

Step 8. Wait 15 seconds. Measure the amount of deflection in the linguine. (Hint: The easiest method for measuring deflection is to use the eraser end of a pencil to line up the deflection of the end of the linguine with its measure on the meter stick.) Record your measurements in the table.

Step 9. Repeat the procedure with two pieces of linguine taped together still hanging 15 centimeters over the edge of the desk. Measure the deflection of the bundle of linguine.

Step 10. Continue repeating the procedure with additional pieces of linguine until you measure deflection with eight pieces taped together. Continue to record your data.

1. Fill in the table with the data you collected.

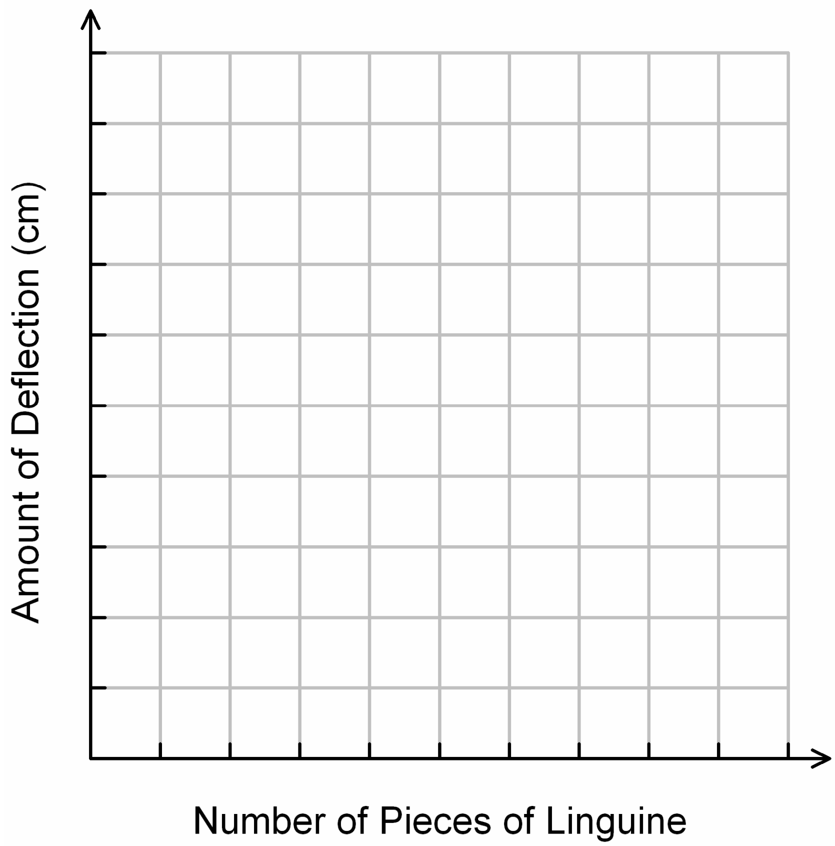
Number of Pieces of Linguine in the Bundle (x)	Starting Height of Linguine Bundle Above the Floor	Height of Linguine Bundle Above the Floor After the Load is Placed	Amount of Deflection in the Linguine (y)	Product of x and y ($x \cdot y$)
1				
2				
3				
4				
5				
6				
7				
8				

2. Write a dependency statement relating the two variables.

3. What is a reasonable domain for the set of data?

4. What is a reasonable range for the set of data?

5. Make a scatterplot of the data you collected.



6. Verbally describe what happens in this data collection investigation.

7. Is this data set continuous or discrete? Why?

8. Does the set of data represent a function? Why?

9. Does the data appear to be a linear, quadratic, exponential or some other type of parent function? Why do you think so?

10. Is the function increasing or decreasing?

11. Is the rate of change constant for this set of data?

12. Determine a function rule that models the set of data you collected.

13. To get a better model, add your set of data to the data of the entire group. Each group should send their data manager to the overhead to fill in the data collected for their group. Record the additional data in the table below. Find the average deflection for each bundle of linguine for the entire group.

Number of Pieces of Linguine in the Bundle	Amount of Deflection for Each Team												Average
	A	B	C	D	E	F	G	H	I	J	K	L	
1													
2													
3													
4													
5													
6													
7													
8													

14. Using the entire group's data, what function would you now use to model this situation?

15. How does this investigation connect to the TEKS from previous courses?

16. What are the key points students need to understand about the Linguine Cantilever before continuing the investigation of rational functions?

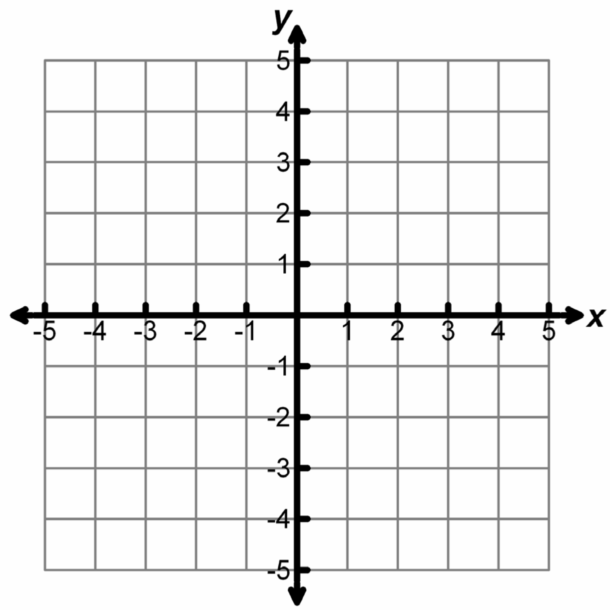
<u>Part 2</u>. Transformations to $f(x)=\dfrac{1}{x}$

1. What is the reciprocal of the linear parent function, $f(x)=x$?

2. Let's investigate some of the attributes of the function and its reciprocal. Fill in the tables with several values for each function. Draw a sketch of the graphs of the two functions on the same set of axes.

$f(x)=x$	
x	y
–3	
–2	
–1	
–0.5	
–0.1	
0	
0.1	
0.5	
1	
2	
3	

$f(x)=\dfrac{1}{x}$	
x	y
–3	
–2	
–1	
–0.5	
–0.1	
0	
0.1	
0.5	
1	
2	
3	



3. Using your graphing calculator (if necessary), fill in the tables below. Let $f(x)=x$ be Y_1, and let $g(x)=\dfrac{1}{x}$ be Y_2.

$$Y_1 = x$$

$$Y_2 = \frac{1}{x}$$

$Y_1 = x$		$Y_2 = \dfrac{1}{x}$
	Intervals where the function is increasing	
	Intervals where the function is decreasing	
	Intervals where the function is undefined	
	Coordinates of the *x*-intercepts (zeros)	
	Equations of any asymptotes	

4. What do you notice about the graphs of the linear parent function and its reciprocal?

5. Where do the linear parent function and its reciprocal intersect?

6. How could you have your students investigate what happens to $f(x)$ as x gets closer and closer to 0 using the graphing calculator?

7. How could you have your students investigate what happens to $f(x)$ as x gets larger and larger?

8. How do the Algebra II TEKS name this new parent function?

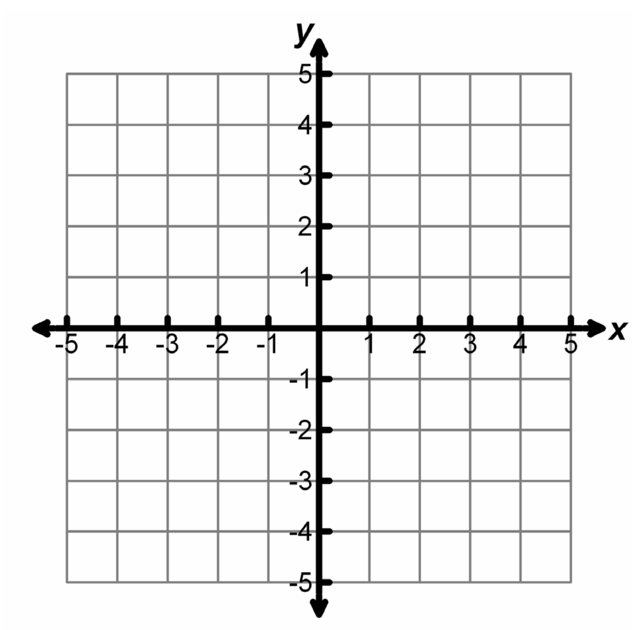
9. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x) = \frac{1}{x}$, when it is multiplied by a constant as in the examples below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results.

$$Y_1 = \frac{1}{x}$$

$$Y_2 = \frac{3}{x}$$

$$Y_3 = \frac{0.1}{x}$$

x	$Y_1 = \frac{1}{x}$	$Y_2 = \frac{3}{x}$	$Y_3 = \frac{0.1}{x}$
−3			
−2			
−1			
0			
1			
2			
3			
4			



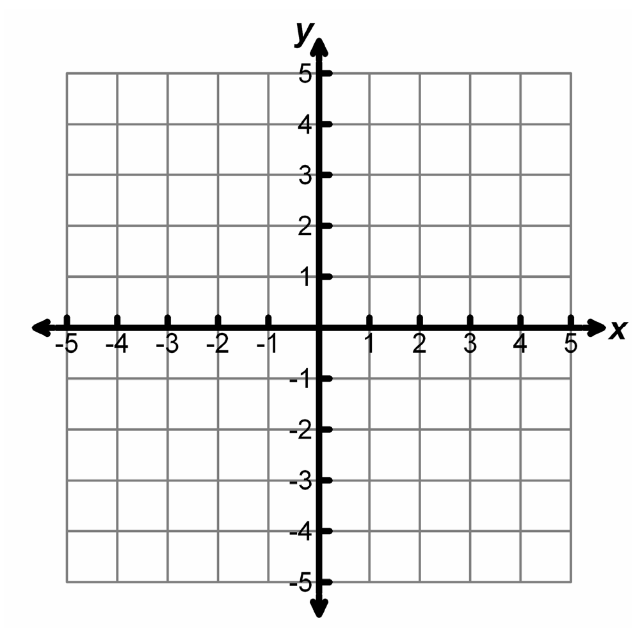
10. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x) = \frac{1}{x}$, when it is multiplied by a negative constant as in the examples below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results which includes any asymptotes.

$$Y_1 = \frac{1}{x}$$

$$Y_2 = -\frac{1}{x}$$

$$Y_3 = -\frac{4}{x}$$

x	$Y_1 = \frac{1}{x}$	$Y_2 = -\frac{1}{x}$	$Y_3 = -\frac{4}{x}$
-3			
-2			
-1			
0			
1			
2			
3			
4			



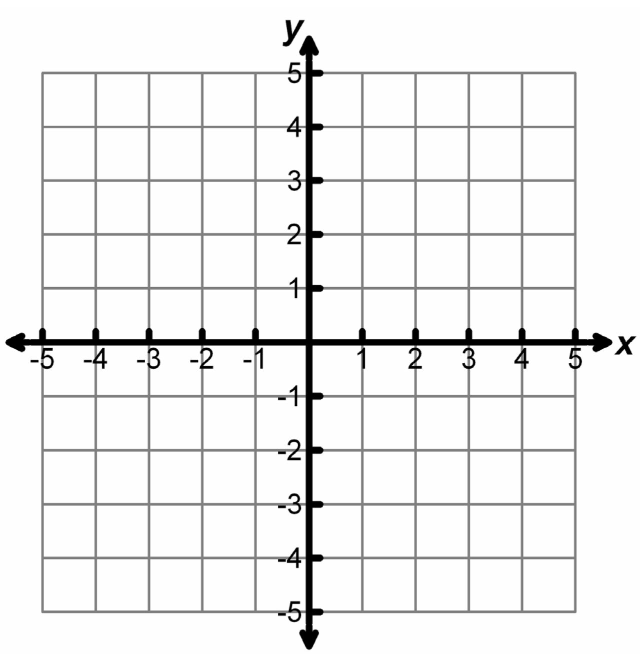
11. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x)=\frac{1}{x}$, if a constant is added to the function, as in the three functions listed below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results which includes any asymptotes.

$Y_1=\frac{1}{x}$

$Y_2=\frac{1}{x}+3$

$Y_3=\frac{1}{x}-2$

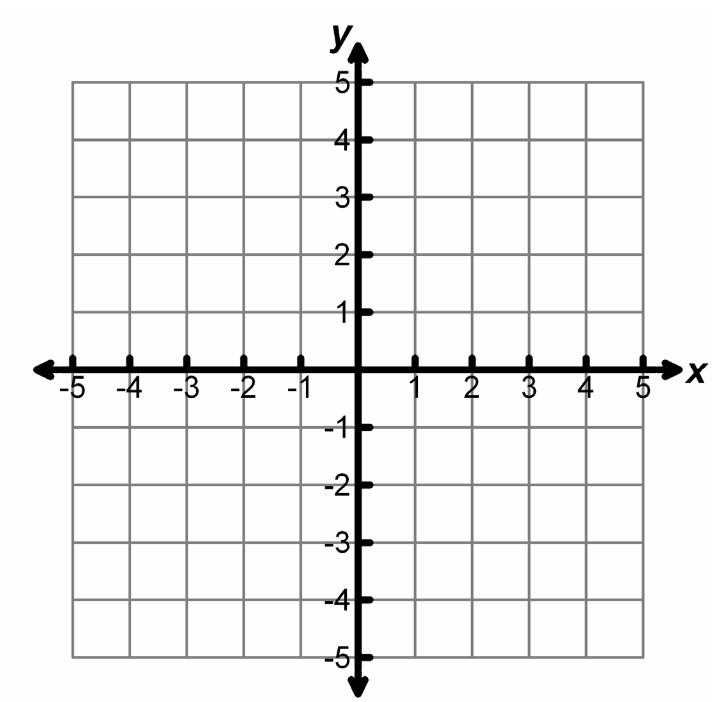
x	$Y_1=\frac{1}{x}$	$Y_2=\frac{1}{x}+3$	$Y_3=\frac{1}{x}-2$
–3			
–2			
−1			
0			
1			
2			
3			
4			



12. Using your graphing calculator, describe what happens to the reciprocal parent function, $g(x)=\frac{1}{x}$, if a constant is added to the x-coordinate in the denominator, as in the three functions listed below. List a few values from the table feature of the graphing calculator in the table below. Show how the transformation is evidenced in your table. Draw a sketch to aid in the description of your results which includes any asymptotes.

$$Y_1=\frac{1}{x}$$
$$Y_2=\frac{1}{x+3}$$
$$Y_3=\frac{1}{x-2}$$

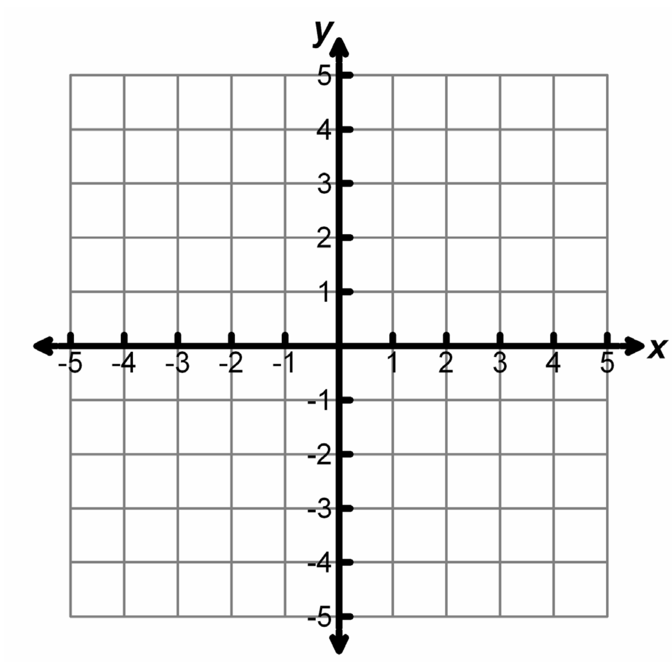
x	$Y_1=\frac{1}{x}$	$Y_2=\frac{1}{x+3}$	$Y_3=\frac{1}{x-2}$
-3			
-2			
-1			
0			
1			
2			
3			
4			



13. Predict, describe and then sketch the transformations to the reciprocal parent function in the function below.

$$f(x) = -\frac{2}{x+1} - 3$$

x	$Y_1 = \frac{1}{x}$	$Y_1 = -\frac{2}{x}$	$Y_1 = -\frac{2}{x+1}$	$Y_2 = -\frac{2}{x+1} - 3$
-3				
-2				
-1				
0				
1				
2				
3				
4				



14. Describe how transformations to the reciprocal function are similar to transformations to other parent functions.

15. What are the key points students need to understand about transformations to the reciprocal function before continuing the investigation of rational functions?

<u>Part 3</u>: Card Sort and Match

1. Sort the first deck of cards into two (and only two) groups. Describe your method for sorting.

2. Using the cards match the rational form of each function to the transformation form or factored form, the equations of any asymptotes, the coordinates of any removable discontinuities (holes), and the graph of the function. Record your answers in the table.

Card Match

Rational Function Form	Transformation Form	Discontinuities, Asymptotes, Other Noteworthy Points	Graph
A			
B			
C			

D			
E			
F			
G			

H			
I			
J			

3. What are the key points students need to understand about rational functions to be able to do the Rational Function Card Match?

<u>Part 4</u>: Length of a Yellow Light

One of the formulas traffic engineers use to help them calculate the length of time a traffic light should remain yellow is

$$Y(t) = t + \frac{v}{2a} + \frac{w+L}{v}.$$

The formula takes into account reaction time, braking time, and intersection clearance time. The variables used in this calculation are:

- t = reaction time (usually 1 second)
- v = velocity of the vehicle (in feet/second)
- a = deceleration rate (approximately 10 feet/ sec^2)
- w = width of the intersection (feet)
- L = length of the vehicle (feet)

In order to calculate the speed limit for a certain intersection that is 48 feet wide, the engineer uses an average car length of 18 feet. She can calculate the length of time the traffic signal should remain yellow at that intersection based on the velocity in feet/second using the following formula:

$$Y(t) = 1 + \frac{v}{20} + \frac{66}{v}.$$

1. If the posted speed limit at the intersection is 55 miles per hour, how long should the signal remain yellow?

<table>
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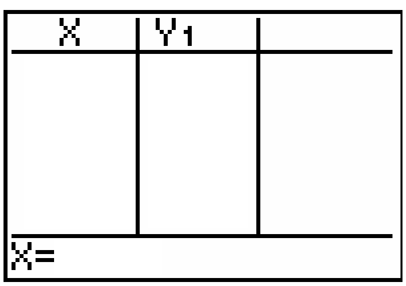
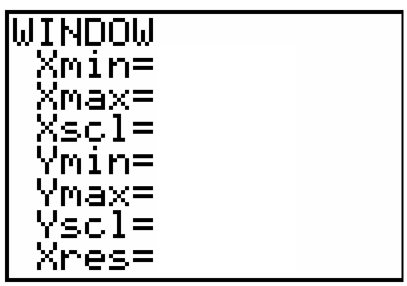
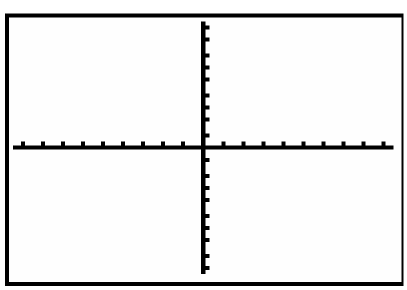

</td>
<td>Solution using a graph:


</td>
<td>Symbolic solution:</td>
</tr>
</table>

2. For a traffic signal to remain yellow for 4 seconds what should the department of transportation post as the speed limit?

3. If the speed of vehicles at a particular intersection varies between 30 and 50 mph, how long do you think the traffic signal should remain yellow?

4. A tractor trailer that is approximately 36 feet long travels through the same intersection when the signal remains yellow for 6 seconds. Based on the formula, how fast should the tractor trailer be allowed to drive through the intersection? How fast should a car be allowed to drive through the intersection?

$$Y(v) = 1 + \frac{v}{20} + \frac{84}{v}$$

5. Do you think this is a good model for length of time that traffic signals should remain yellow for every x value in the domain?

Bonus question:

6. What is the oblique asymptote for this rational function? Graph both the function and the asymptote on your graphing calculator.

$$Y(v) = 1 + \frac{v}{20} + \frac{66}{v}$$

7. Do you think that students should solve every problem involving rational functions symbolically? What understanding would students gain from solving with tables and graphs?

Watch This!

5E Instructional Model Phases	The teacher…	The student…	Evidence of learning…
Engage			
Explore			
Explain			
Elaborate			
Evaluate			

Debrief This!

1. How does the teacher set the stage for the lesson?

2. How does the teacher relate new contexts to students' prior knowledge?

3. How does the teacher place the responsibility on the students for the lesson?

4. What is the teacher doing that you might not normally see in an Algebra II classroom?

5. What student behavior do you see in the Explore that you might not normally see in an Algebra II classroom?

6. How is the Evaluate different from what you might normally see in an Algebra II classroom?

7. How does student behavior change as the 5E lesson progresses? Why do you think it changes?

What Next?

1. What topics comprise the next unit of instruction for your students?

2. What topics within this unit present challenges for your students as they learn? Why?

3. What opportunities for modeling exist in these challenging topics?

4. What opportunities for organizing exist in these challenging topics?

5. What opportunities for generalizing exist in these challenging topics?

6. What five questions do you want your students to be able to answer related to modeling, organizing, and generalizing within this unit?

7. How might facilitating connections among the processes of modeling, organizing, and generalizing help your students be more successful in this unit of study?