Andy and Beca

Andy and Beca are renting videos for the weekend. They can only to afford to rent a maximum of six videos. Some of the videos must be on VHS tapes and some must be on DVD.

1. What are the possible combinations of VHS and DVD Andy and Beca can rent? Use the table to list all the possible combinations.

VHS	DVD

Open **Video Rental Sketch 1** through TI Interactive.

2. What does each plotted point represent on the graph?

3. Why are there no points with negative coordinates plotted on the graph?

4. Does your table match the table shown on the sketch?

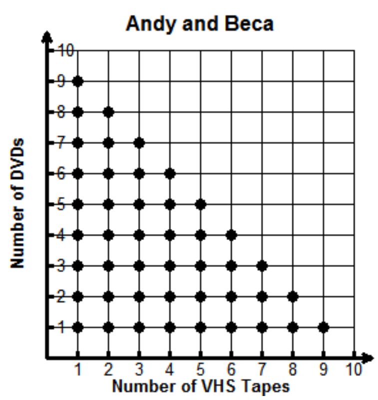
5. Predict how the graph would change if Andy and Beca could rent a total of 10 videos.

6. Open **Video Rental Sketch 2** to check your prediction. How do the two graphs compare? Explain.

7. What are the total number of combinations of renting VHS tapes and DVDs?

8. If Andy and Beca limit the number of VHS tapes to 5 or less and the number of DVDs to 7 or less, how would the graph change?

9. Shade the graph below to show the new restrictions to the number of VHS tapes and DVDs Andy and Beca could rent.



10. What are the possible combinations, with the new restrictions included?

11. What are the outermost points of the restricted region?

12. If VHS tapes rent for $4 and DVDs rent for $2, what is the most they could spend if they stay within all the restrictions? What combination of VHS tapes and DVDs would that be?

13. If VHS tapes rent for $3 and DVDs rent for $4, what is the most they could spend? What combination of VHS tapes and DVDs would that be?

14. How did the cost change from the first situation to the second situation? Why?

Video Joe

Video Joe has decided to open a small video rental store. He plans on offering DVDs and VHS tapes for rental. After installing all the shelves in the store, he calculates that he has 125 feet of shelf space to store the DVDs and VHS tapes. Each DVD takes up 5 inches of shelf space, while each VHS tape takes up 4 inches of shelf space.

Let x = the number of DVDs and y = the number of VHS tapes he can stock on his shelves at any given time.

1. Write an equation describing the number of VHS tapes and DVDs Video Joe can stock on his shelves, given the limited amount of shelf space.

2. Would Video Joe be able to stock more or less VHS tapes and DVDs than represented by the equation? Justify your answer.

3. Write the equation as an inequality to represent this situation.

4. Write the inequality in slope – intercept form.

5. Graph this inequality on TI Interactive or graphing calculator. Describe the region that would apply to this inequality.

6. Video Joe would like to keep at most 2 times as many VHS tapes as DVDs. Write an inequality to represent this restriction.

7. Graph this inequality on TI Interactive or graphing calculator on the same screen as the previous inequality. Describe the region that now applies to the two restrictions (inequalities).

8. Video Joe would also like to keep between 80 and 200 VHS tapes in stock. Write two inequalities to represent this restriction.

9. Graph these two inequalities on TI Interactive or graphing calculator on the same screen as the two previous inequalities. What region represents all the restrictions, or inequalities, in this situation?

10. Open **Video Rental Sketch 3**. How does your graph compare to this one? Explain. (The purple trapezoidal region represents the region common to all restrictions.)

11. What are the vertices of the region common to all the restrictions (**feasible region**)?

12. What do these coordinates represent in this situation?

13. Video Joe makes a profit of $2.25 on each DVD rented and $1.50 on each VHS tape rented. Write a function representing the profit he makes if he rents x number of DVDs and y number of VHS tapes.

14. Use the profit function to determine the amount of profit Video Joe would make using the coordinates of the feasible region.

15. Use the spreadsheet in TI Interactive to enter the coordinates of the feasible region and the profit function. Open **Video Rental Spreadsheet 1** to verify your answers. How do these answers compare with yours? Explain any differences.

16. Which combination would generate the most profit for Video Joe, but still meet all the restrictions? How do you know?

Shipping Costs

Video Joe orders all his DVDs and VHS tapes from an area supplier. The supplier has only one truck available for delivery and it has a capacity of 3600 cubic feet. One case of VHS tapes takes up 18 cubic feet of space, while one case of DVDs takes up 12 cubic feet of space. Video Joe places an order with the supplier for one truckload of VHS tapes and DVDs. He has to order between 150 and 240 cases of DVDs to meet the demand and at least 20 cases of VHS tapes. The shipping costs are based on the number of cases on the truck. Each case of VHS tapes costs $3.50 in shipping costs and each case of DVDs costs $3.75 in shipping costs.

Let x = number of cases of VHS tapes
y = number of cases of DVDs

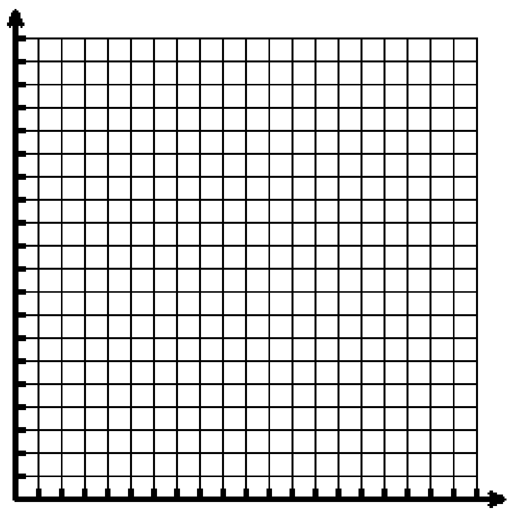
How many cases of VHS tapes and DVDs should he order if he would like to pay the least amount possible in shipping costs and stay within all the restrictions?

Casey's Part-time Jobs

Casey the college student is working two part-time jobs. He works at a video rental store for $5.25 per hour and at a movie theatre for $6.05 per hour. He wants to work no more than 30 hours per week. He wants to work between two and three times the hours at the movie theatre than at the video store. He also has to work a minimum of 10 hours per week at the movie theatre.

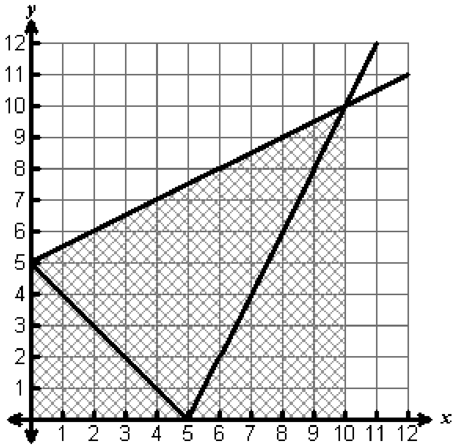
Let x = the number of hours worked at the video store
y = the number of hours worked at the movie theatre

Use TI-Interactive or a graphing calculator to graph the feasible region described above. Record the feasible region below. Label the axes and the vertices of the feasible region.



How many hours should he work at each job to earn the maximum amount of money each week? What is the maximum amount of money he could make each week? Justify your answers.

1 Shown below is a feasible region. The profit function for the region is $f(x, y) = 6x + 5y$.



What are the minimum and maximum values of the function?

A 5 and 10

B 25 and 110

C 25 and 30

D 0 and 110

2 A company machines and sells nuts and bolts. One machine can produce 100 nuts and 50 bolts each hour. Due to demand, the company must produce at least two times more nuts than bolts. The machine runs a maximum of 40 hours per week. Which set of inequalities shows the correct restrictions for this situation if x represents the number of nuts produced in one week and y represents the number of bolts produced in one week?

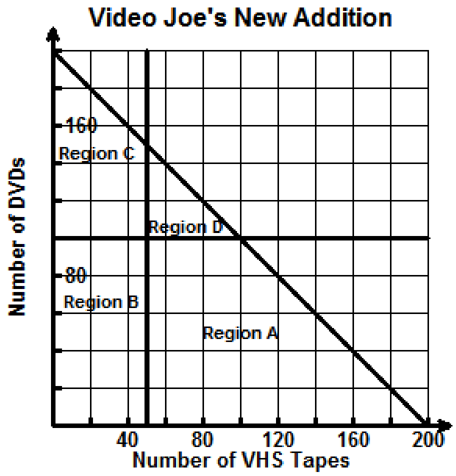
A $\frac{x}{100} + \frac{y}{50} \leq 40$
$y \geq \frac{1}{2}x$

B $\frac{x}{50} + \frac{y}{100} \leq 40$
$y \geq \frac{1}{2}x$

C $\frac{x}{100} + \frac{y}{50} \leq 40$
$y \geq 2x$

D $\frac{x}{50} + \frac{y}{100} \leq 40$
$y \geq 2x$

3 Video Joe is expanding his video store. He added enough shelving to hold a maximum of 200 items. He wants to have 50 VHS tapes at most and at least 100 DVDs in stock at all times in the new addition.



Which of the following regions represents the limited restrictions of this situation?

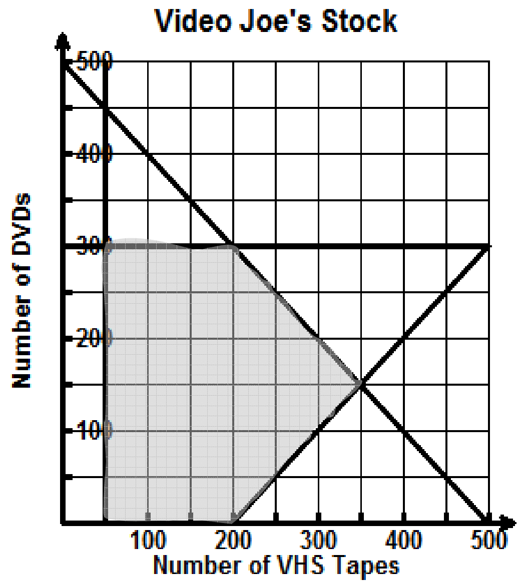
A Region A

B Region B

C Region C

D Region D

4 The feasible region shown below represents the possible amounts of VHS tapes and DVDs on Video Joe's shelves at any given time.



If he makes $1.75 on each VHS rental and $2.00 on each DVD rental, which combination of VHS tapes and DVD rentals would result in the most profit?

A 350 VHS and 150 DVD

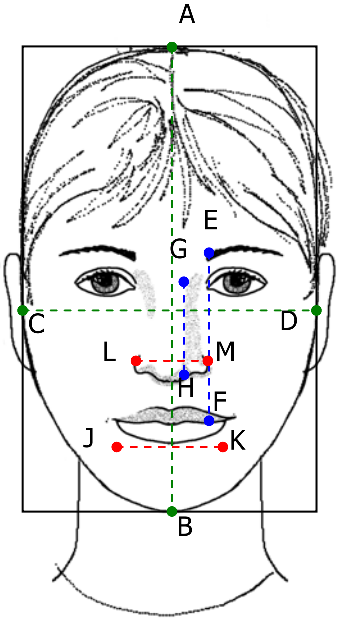
B 200 VHS and 300 DVD

C 150 VHS and 350 DVD

D 300 VHS and 200 DVD

The Eye of the Beholder

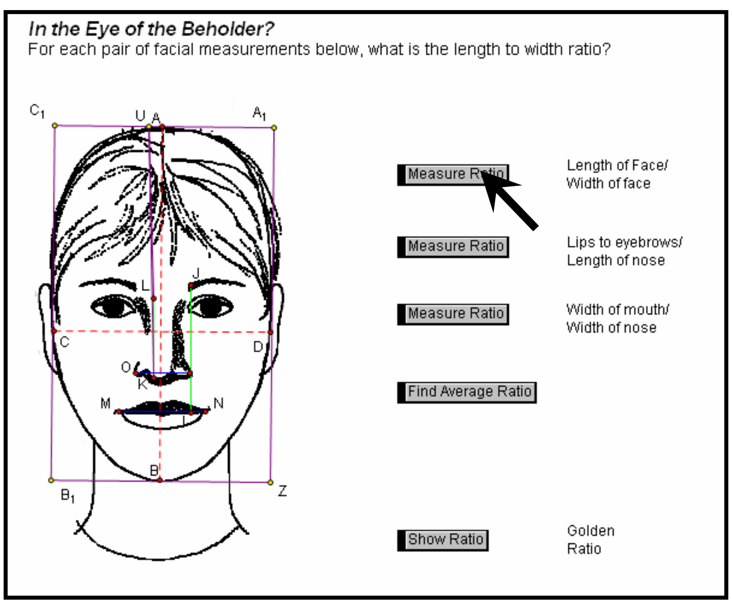
1. Study the features on the artist's sketch below. Identify the segments that represent each of the following ratios.



Sketch by artist, Debra L. Hayden, 2005.
Used by permission

Ratio	Segments
Length of face to Width of face	Length of face / Width of face =
Lips to eyebrows to Length of nose	Lips to eyebrows / Length of nose =
Width of mouth to Width of nose	Width of mouth / Width of nose =
Average Ratio	

2. Open the sketch **Face Sample** in Geometer's Sketchpad. Calculate the ratio values indicated in the sketch by clicking on the "Measure Ratio" action button for the given ratio. Also, calculate the average ratio. Record your answers in your table.



3. Log on to the Internet and open the website http://www.angelfire.com/celeb2/celebrityfaces/. Search for a photo of your favorite celebrity. The photo must be a full front view of the face.

4. Right click on the face and select “Copy” so that you can “insert” the photo into Geometer’s Sketchpad.

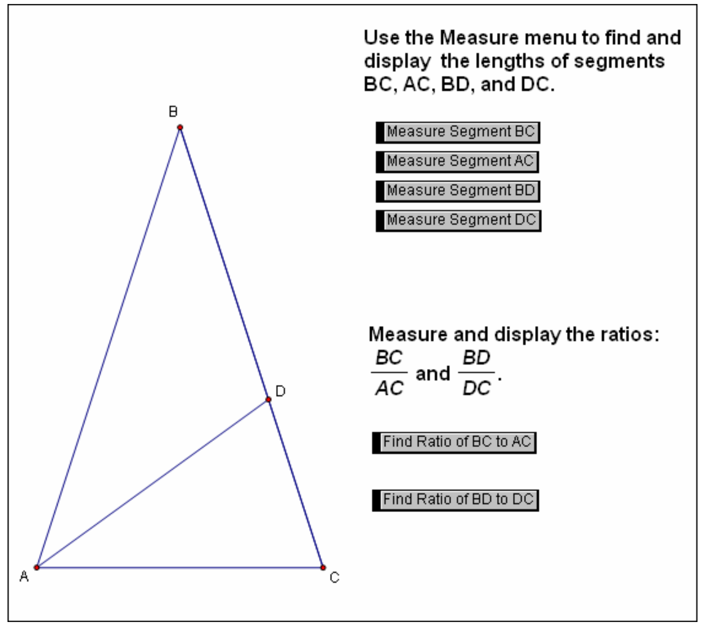
5. Using Geometer’s Sketchpad, construct and measure segments of the face you copied as shown on the sample. Measure the appropriate ratios and record them in the chart below.

<table>
<tr><td colspan="4">I used a photo of :</td></tr>
<tr><td>Length of face</td><td></td><td rowspan="2">Ratio</td><td rowspan="2"></td></tr>
<tr><td>Width of face</td><td></td></tr>
<tr><td>Lips to eyebrows</td><td></td><td rowspan="2">Ratio</td><td rowspan="2"></td></tr>
<tr><td>Length of nose</td><td></td></tr>
<tr><td>Width of mouth</td><td></td><td rowspan="2">Ratio</td><td rowspan="2"></td></tr>
<tr><td>Width of nose</td><td></td></tr>
</table>

6. How do your ratios compare with those found by other groups in the class? Why do you think this is so?

Creating a "Golden" Exponential Function

1. Open the sketch **golden triangle1** to find possible measurements for each of the following:



The length of $\overline{BC}$ = ______________

The length of $\overline{AC}$ = ______________

The ratio of $\overline{BC} : \overline{AC} \approx$ ______________

The length of $\overline{BD}$ = ______________

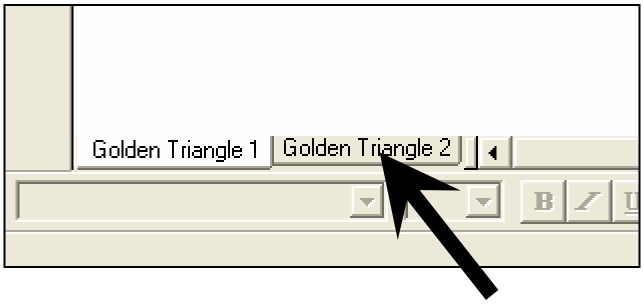
The length of $\overline{DC}$ = ______________

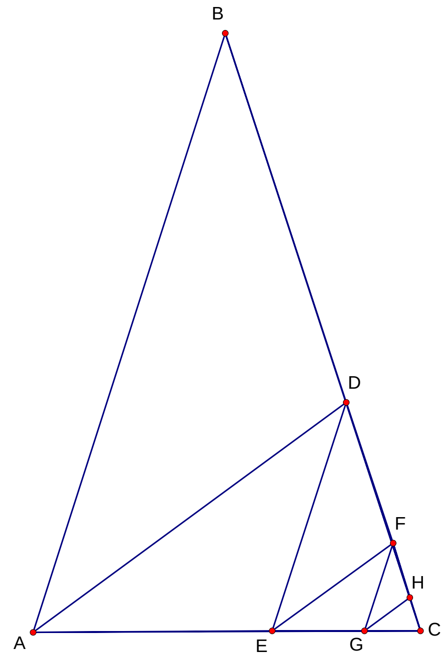
The ratio of $\overline{BD} : \overline{DC} \approx$ ______________

2. Click on Point *C* and drag it around the screen. What happens to the segment lengths?

3. What happens to the ratios when you drag point *C* around the screen?

4. Click the **Golden Triangle 2** tab inside your sketch. Find possible values for the following:





Triangle	Leg	Length	Successive Ratios
1	$\overline{HC}$		
2	$\overline{GC}$		
3	$\overline{FC}$		
4	$\overline{EC}$		
5	$\overline{DC}$		
6	$\overline{AC}$		
7	$\overline{BC}$		

5. Enter the numbers 1 – 7 into List 1 on your graphing calculator. Enter the lengths of the segments $\overline{HC}$, $\overline{GC}$, $\overline{FC}$, $\overline{EC}$, $\overline{DC}$, $\overline{AC}$, and $\overline{BC}$ into List 2. Create a scatter plot on your graphing calculator with the Triangle number on the *x*-axis and the Leg length on the *y*-axis. Sketch your plot and describe your window.

6. Determine an exponential function that passes through these points. Explain how you determined the function.

7. Sketch your plot and function graph. Does the function fit the data well? How do you know?

8. What does the coefficient in your function represent in the golden triangle? How did you obtain this value?

9. What does the base of the power in your function represent in the golden triangle?

Algebra and the Golden Ratio

You have found the exact value of the golden ratio to be $\frac{1+\sqrt{5}}{2}$. Let's look at how this value connects to the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, …

Consider the table below but don't fill in the right-hand column until you've answered questions 1 – 3.

Term number	Fibonacci number
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

1. If you made a scatter plot of Fibonacci number vs. term number, what would the scatter plot look like?

2. If you started with 1 as your first Fibonacci number, could you write a function that would pass through all of the points in your scatter plot?

3. How could you make a scatter plot that more closely fits an exponential function?

4. Fill in the table with the Fibonacci numbers of your choice and write an exponential function to fit your points.

5. Which would give a better fit: starting with 5 or starting with 13? How does choosing a different starting number affect your function rule?

The Golden Ratio in Art and Architecture

Search the Internet using key words “golden ratio” and “art” or “architecture.” Find one example of how the golden ratio is used in art and one example of its use in architecture. Record at least the following information for each example.

Art Example

The artist is/was ______________________________

The name of the painting, sculpture, etc. is ____________________

Give a brief description or simple sketch of how the golden ratio is used in this work.

Architecture Example

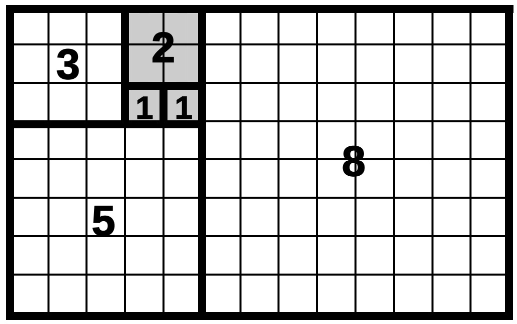
The architect is/was ___________________________(or give country where it is located)

The name of the painting, sculpture, etc. is ____________________

Give a brief description or simple sketch of how the golden ratio is used in this structure.

Golden Areas

Consider the squares that make up a golden rectangle shown below. The squares have sides that are terms of the Fibonacci sequence: 1, 1, 2, 3, 5,… Each golden rectangle, such as the square that is shaded, is formed by attaching the next Fibonacci square to the previous golden rectangle.



1. Complete the table below to show the relationship between the number of a square in each golden rectangle and the area of the square. Let the 3 × 3 square be square #1.

Square Number	**Area of Square**
1	3^2
2	5^2
3	
4	
5	
6	
7	

2. Enter the square numbers into List 1 of a graphing calculator and the areas into List 2. Make a scatter plot for squares 1 – 7. Sketch your scatter plot below and describe the domain and range of the plot.

3. Without using the regression feature on your calculator, write a function that fits your data. Enter your function into your calculator to test it. Alter the function as needed until you are satisfied that it fits the data.

4. Explain how the numbers in your function are related to the data.

5. Would your function be any different if you started with 2^2 instead of 3^2 as the first area? If so, how and why?

1 A stationery company makes cards and posters using dimensions of golden rectangles. So far their inventory includes posters with dimensions (in inches) of 3×5, 5×8, and 8×13. Which equation below would be useful in approximating the length of a poster with a width of 21 inches?

A $L = 13 \times 21$

B $L = 3 \times 1.6^4$

C $L = 13 + 21$

D $L = (1.6)(21)$

2 The table below shows a section of the Fibonacci sequence.

Term number *x*	**Fibonacci number *y***
0	5
1	8
2	13
3	21
:	:

Which function best fits the data shown in the table?

A $y = 1.6x$

B $y = 5 * 1.6^x$

C $y = x^{1.6}$

D $y = 8 * 1.6^x$

3 The exact value of *phi*, referred to as the golden ratio, can be found by taking the larger root of the equation $x^2 = x + 1$. What is the exact value of *phi*?

A $\frac{5}{3}$

B 1.618

C $\frac{1+\sqrt{5}}{2}$

D $\frac{1-\sqrt{5}}{2}$

4 The function $y = 2(1.62)^x$ produces the table below when the domain is {1, 2, 3, …}.

X	Y1	
1	3.24	
2	5.2488	
3	8.5031	
4	13.775	
5	22.315	
6	36.151	
7	58.565	

X=1

Which function will produce the table

X	Y1	
1	8.5031	
2	13.775	
3	22.315	
4	36.151	
5	58.565	
6	94.875	
7	153.7	

X=1

for the same domain?

A $y = 1.2346 * 1.62^x$

B $y = 3.24 * 1.62^x$

C $y = 5.2488 * 1.62^x$

D $y = 8.5031 * 1.62^x$